M.Sc. Third Semester (Mathematics) (CBCS) NEP -

MMT3T10 Compulsory Paper-I - M10 - Complex Analysis

P. Pages : 2
Time : Three Hours

SKR/KW/24/10244
Max. Marks : 80

Notes : 1. Solve five questions. Choosing one from each unit and Question no. 9 is compulsory.

UNIT – I

1\. a) Find the radius of convergence of the following power series: **8**

i) $\sum \frac{(n!)^2 z^n}{(2n!)}$ ii) $\sum \frac{n!}{n^n} Z^n$

b) Let $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ have radius of convergence $R > 0$, then show that **8**

i) for each $k \geq 1$ the power series $\sum_{n=k}^{\infty} n(n-1)....(n-k+1) a_n (z-a)^{n-k}$ has radius of convergence.

ii) The function f is infinitely differentiable, then $f^k(z)$ is given by the power series $\sum_{n=k}^{\infty} n(n-1)......(n-k+1) a_n (z-a)^{n-k}$, for all $k \geq 1$ and $|z-a| < R$.

OR

2\. a) Define complex number and explain 'The complex Plane' in detail. **8**

b) If $\Sigma a_n (z-a)^n$ is a given power series with radius of convergence R, then show that $R = \lim \left| \frac{a_n}{a_{n+1}} \right|$, if this limit exist. **8**

UNIT – II

3\. a) If S is a Mobius transformation, then show that S is composition of translation, dialation and the inversion (of course, some of these may be missing). **8**

b) Let z_1, z_2, z_3, z_4 be four distinct points in C_∞ then (z_1, z_2, z_3, z_4) is a real number if and only if all four points lie on a circle. **8**

OR

4\. a) Let $\phi : [a,b] \times [c,d] \to C$ be a continuous function and define $g : [c,d] \to C$ by $g(t) = \int_a^b \phi(s,t)\, ds$, then show that g is continuous, Moreover $\frac{\partial \phi}{\partial t}$ exist and is continuous function on $[a, b] \times [c, d]$, then show that g is continuous differentiable and **8**

$$g'(t) = \int_a^b \frac{\partial \phi(s,t)}{\partial t} \cdot ds.$$

b) If $\gamma : [0,1] \to C$ is a closed rectifying curve and $a \notin \{\gamma\}$, then show that $\frac{1}{2\pi i} \int_\gamma \frac{dz}{z-a}$ is an integer. **8**

UNIT – III

5. a) Let γ be rectifiable curve and suppose ϕ is a function defined and continuous on $\{\gamma\}$, for each $m \geq 1$. Let $F_m(z) = \int_\gamma \phi(w)(w-z)^{-m} dw$, for $z \notin \{\gamma\}$, then show that F_m is analytic on $c - \{\gamma\}$ and $F'_m(z) = mF_{m+1}(z)$. 8

b) Let G be simply connected and let $f: G \to C$ be an analytic function such that $f(z) \neq 0$, for any $z \in G$, then there exist an analytic function $g: G \to C$ such that $f(z) = \exp(g(z))$ if $z_0 \in G$ and $e^{W_0} = f(z_0)$, then show that we may choose g such that $g(z_0) = W_0$. 8

OR

6. a) State and prove Argument Principle. 8

b) If P(z) is a non-constant polynomial then show that there is a complex number with $P(z) = 0$. 8

UNIT – IV

7. a) State and prove Schwarz's Lemma. 8

b) Define Convex set and also show that a function $f:[a,b] \to R$ is convex, if the set $A = \{(x,y) / a \leq x \leq b \text{ and } f(x) \leq y\}$ is convex. 8

OR

8. a) Show that a differentiable function f on [a, b] is convex if and only if f' is increasing. 8

b) State and prove Hadamards Three Circle theorem. 8

Compulsory Question

9. a) Let $\sum a_n$ and $\sum b_n$ be two absolutely converging series and put $c_n = \sum_{k=0}^{n} a_k b_{n-k}$, then show that $\sum c_n$ is absolutely convergent with sum $(\Sigma a_n)(\Sigma b_n)$. 4

b) If z_2, z_3 and z_4 are distinct points and T is any Mobius Transformation, then $(z, z_2, z_3, z_4) = (Tz, Tz_2, Tz_3, Tz_4)$, for any point z. 4

c) Let G be a region and suppose that f is non-constant analytic function on G. Then show that for any open set U in G, f (U) is open. 4

d) Let G be region in $\mathbb{C}$ and f be an analytic function on G. Suppose there is a constant M such that $\lim_{z \to a} \sup |f(z)| \leq M$, for all a in $\partial_\infty G$, then $|f(z)| \leq M$, for all z in G. 4

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M. Sc. Third Semester (Mathematics) (CBCS) NEP

Compulsory Paper-II MMT3T11 M11: Functional Analysis

P. Pages : 2 **SKR/KW/24/10245**

Time : Three Hours Max. Marks : 80

Note : Solve **five** questions, choosing **one** from each unit. Question No. 9 is compulsory.

UNIT-I

1. a) State and prove Riesz's Lemma. 8

   b) If X is finite dimension norm space, then prove that a subset M of X is compact iff M is closed and bounded. 8

OR

2. a) Define equivalent norms and prove that on a finite dimensional vector space, any two norms are equivalent. 8

   b) Every compact subset of a metric space is closed and bounded, but converse is not true. 8

UNIT-II

3. a) Prove that a finite dimension vector space is algebraically reflexive. 8

   b) State and prove Schwarz's Inequality. 8

OR

4. a) Prove that R^n is a Hilbert Space. 8

   b) Prove that a dual space X' of a norm space is always Banach Space. 8

UNIT-III

5. a) State and prove Riesz's Representation theorem. 10

   b) Prove that inner product space $(X, <\cdot>)$ is bounded sequi-linear form. 6

OR

6. a) Define Hilbert space Let $H_1 \& H_2$ be Hilbert space and let $S: H_1 \to H_2$ & $T: H_1 \to H_2$ be a bounded linear operations, then prove the following, 8

   i) $\langle T_y^*, x \rangle = \langle y, T_x \rangle$

   ii) $(S+T)^* = S^* + T^*$

   iii) $(\alpha T)^* = \bar{\alpha} T^*$

iv) $(T^*)^* = T$

v) $\|T^*.T\| = \|TT^*\| = \|T\|^2$

vi) $T^*T = 0$ iff $T = 0$

vii) $(ST)^* = T^*S^*$

b) Let $T: H \to H$ be bounded linear operator on Hilber Space H, then prove the following. 8

i) If T is self adjoint then $\langle T_x, x \rangle$ is real $\forall x \in H$.

ii) If H is complex and $\langle T_x, x \rangle$ is real, then T is self adjoint $\forall x \in H$.

UNIT-IV

7. a) State and prove open mapping theorem. 6

b) Show that every complete metric space is not meager. 10

OR

8. a) State and prove closed graph theorem. 8

b) State and prove uniform Bohdedness Theorem. 8

Compulsory Question

9. a) Show that on a finite dimension norm space, every linear operator is bounded. 4

b) Prove that C[a, b] is not an inner product space, by showing that its norm does not satisfy parallelogram law. 4

c) Define: 4

i) Isometric operator

ii) Normal operator

iii) Self adjoint operator

iv) Unitary operator

d) Define weak Cauchy sequence in a normed space and prove that it is bounded. 4

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M. Sc. Third Semester (Mathematics) (CBCS) NEP

Compulsory Paper-III : MMT3T12 M12 : Advanced Mathematical Methods

P. Pages : 3 | SKR/KW/24/10246

Time : Three Hours | Max. Marks : 80

Notes :
1. Solve five questions choosing one from each unit.
2. Question No. 9 is compulsory.

UNIT – I

1. a) Find the Fourier series of the function defined as, 8

$$f(x)=\begin{cases} x+\pi & \text{for } 0<x<\pi \\ -x-\pi & \text{for } -\pi<x<0 \end{cases} \text{ and } f(x+2\pi)=f(x).$$

b) Obtain the Fourier cosine series expansion of the periodic function defined by- 8

$$f(t)=\sin\left(\frac{\pi t}{\ell}\right),\ 0<t<\ell$$

OR

2. a) 8

$$\text{If } f(x)=\begin{cases} \pi x & , \quad 0<x<1 \\ \pi(2-x), & 1<x<2 \end{cases}$$

using half range cosine series, show that:

$$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+........=\frac{\pi^2}{96}$$

b) Find the Fourier series expansion of the periodic function of period 2π defined by, 8

$$f(x)=\begin{cases} x & , \text{if } -\pi/2<x<\pi/2 \\ \pi-x, & \text{if } \pi/2<x<3\pi/2 \end{cases}$$

UNIT – II

3. a) Let f(t) and g(t) be two continuous functions of positive variables and $L[f(t)]=\bar{f}(p)$ and $L[g(t)]=\bar{g}(p)$ then, $L[(f*g)(t);p]=\bar{f}(p)\cdot\bar{g}(p)$. 8

b) Find the Laplace transform of: 8

i) $t^2\cdot\cos at$ ii) $\frac{\sin at}{t}$

OR

4. a) Using Laplace transforms, solve the differential equations. 8

$$(D+1)y_1+(D-1)y_2=e^{-t},$$

$$(D+2)y_1+(D+1)y_2=e^{t}$$

Where, $D=\frac{d}{dt}$ and $y_1(0)=1, y_2(0)=0$.

8

b) Solve; $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=e^{-x}\cdot\sin x$.

Where $y(0)=0,\ y'(0)=1$.

UNIT – III

5. a) State and prove Fourier Integral Theorem and express it in exponential form. 8

b) Solve the Laplace equation $\Delta_2 u=0,\ 0\le y\le a$ with boundary condition 8

$u(x,0)=f(x)$ and $u_y(x,a)=0$

OR

6. a) Using Parseval's identity, prove that 8

$$\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)}=\frac{\pi}{2ab(a+b)}$$

b) Find the solution in $D=\{(x,y): 0\le x\le a,\ 0\le y\le b\}$. If the diffusion equation, 8

$K\Delta_2 u(x,y,t)=\frac{\partial u}{\partial t},\ t>0$ where, u vanishes on boundary of D and

$u(x,y,o)=f(x,y),\ (x,y)\in D$.

UNIT – IV

7. a) Define Z-transform. Find the Z-transform of $\sin(\alpha k),\ k\ge 0$. 8

b) If $\{f(k)\}=F(Z),\ \{g(k)\}=G(Z)$, a and b are constants, then prove that; 8

$Z^{-1}[aF(Z)+bG(Z)]=a\ Z^{-1}[F(Z)]+b\ Z^{-1}[G(Z)]$

And hence find the inverse Z-transform of $\frac{1}{Z-2}$.

OR

8. a) Obtain : $Z^{-1}\left[\frac{2Z^2-10Z+13}{(Z-3)^2(Z-2)}\right]$, when $2<|Z|<3$. By Partial fraction method. 8

b) Solve the difference equation: 8

$6y_{k+2}-y_{k+1}-y_k=0,\ y(0)=0,\ y(1)=1$

by Z-transform.

8 Compulsory Question.

a) Expand $f(x) = e^{x}$ in a cosine series over (0, 1). 4

b) Obtain: $L^{-1}\left[\dfrac{1}{s\left(s^{2}+a^{2}\right)}\right]$ 4

c) If F(s) is the complex Fourier transform of f(x), then $F\{f(ax)\} = \dfrac{1}{a}F\left(\dfrac{s}{a}\right)$ 4

d) Prove that: $\lim\limits_{K\to\infty} f(k) = \lim\limits_{Z\to 1} (Z-1)\cdot F(Z)$ 4

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M. Sc. Third Semester (Mathematics) (CBCS) NEP

Elective(A) Optional Paper-IV : MMT3T13 M13: General Theory of Relativity

P. Pages : 2 SKR/KW/24/10247

Time : Three Hours Max. Marks : 80

Note : 1. Solve all five questions. Choosing One from each of the four units.
2. Question No. 9 is compulsory.

UNIT – I

1. a) Define Outer product and Inner Products. 8
Let A^r be an arbitrary contravariant vector if the inner product $A^r B_r$ is invariant then B_r is covariant vector.

b) State and prove Bianchi Identity. 8

OR

2. a) Show that Einstein tensor has zero divergence. 8

b) Using variational principle derive differential equation of geodesic in Riemannian space. 8

UNIT – II

3. a) Obtain the differential equation of Geodesic for the metric, 8
$$ds^2 = f(x)dx^2 + dy^2 + dz^2 + \frac{1}{f(x)}dt^2$$

b) Discuss in detail the principle of equivalence and principle of covariance. 8

OR

4. a) Explain in detail Mach Principle. 8

b) Prove that the filed equations of general relativity can be recovered from the Poisson's equation of Newtonian theory of gravitation. 8

UNIT – III

5. a) Discuss in brief the Bending of light rays. 8

b) State and Prove Birkhoff's theorem. 8

OR

6. a) Obtain the differential equation of planetary orbit. 8

b) Discuss Schwarzschild's exterior solution in a isotropic form. 8

UNIT – IV

7. a) Find the interior solution when pressure is same everywhere in spherically symmetric body i.e. $p = p_0$ = constant. 8

b) Write the expression of pressure in the Newtonian limit. 8

OR

8. a) Explain the expression of Tolman – Oppenheimer – Volkoff equation. 8

b) Derive the expression of Schwarzschild's Interior Solution. 8

9. Compulsory question.

a) Derive the relation between absolute derivative and covariant derivative of a contravariant vector field. 4

b) Find the non-vanishing Christoffel symbols of the given metric, 4

$$ds^2 = -e^{2Rt}\left(dx^2 + dy^2 + dz^2\right) + dt^2$$

c) Discuss the Schwarzschild Singularity. 4

d) Find the interior solution for spherically symmetric body when pressure p and density ρ are related as $p = -\rho$. 4

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