PSM/KW/23/10007

Master of Science (M.Sc.) Mathematics Semester—I (CBCS) (NEP) Examination

MMT1T05 M5 : RESEARCH METHODOLOGY IN MATHEMATICS

Paper—5

Time : Three Hours] [Maximum Marks : 60

N.B. :— Solve **one** question from each unit. Question no. **9** is compulsory.

UNIT–I

1. (a) What is "Research Process"? Explain in brief the different steps involved in the Research process. 6

   (b) Outline the main objectives of research and provide detailed explanation for each objective. 6

OR

2. (a) What is a Research Problem ? Discuss the relation between research problem and choice of research design. 6

   (b) What are the problems usually faced by researchers in India? Explain. 6

UNIT–II

3. (a) Write short note on Problem and Project-based learning. 6

   (b) Explain the various steps involved in the research project work process. 6

OR

4. (a) List and explain in short, the contents of the general structure of the project report. 6

   (b) What are the ethical issues concerning the researcher's research activity ? Explain. 6

UNIT–III

5. (a) What is Patent ? Describe in detail, its process for granting the status and uses for innovators. 6

   (b) Describe 'copyright' and the work protected under Copyright Act. Explain the process of obtaining copyright. 6

OR

6. (a) What is TradeMark ? Explain the different types of trademarks with examples. 6

   (b) What is a Geographical Indication (GI) ? Name the legislation for its protection in India and briefly out line the procedure of registration. 6

UNIT–IV

7. (a) Describe/ List the advance methods to search the required information effectively. 6

   (b) Discuss about (i) LaTeX and (ii) Microsoft word, clearly pointing out the difference between these paper formating softwares. 6

OR

8. (a) What do you mean by research paper ? How can we create a research paper in LaTeX? Explain. 6

   (b) What is 'Plagiarism' in research? Explain, how plagiarism can be avoided in research? 6

9. **(Compulsory)**

   (a) Write a short note on 'Review of Literature' in research. 3

   (b) Describe various roles of the group member for best performance in the group process learning. 3

   (c) Explain in short, the necessary conditions for grant of patent to the invention. 3

   (d) Describe various reference styles used in academic writing. 3

Master of Science (M.Sc.) Semester—I (Mathematics) (CBCS) Examination

INTEGRAL EQUATIONS (OLD)

Paper — 5

Paper — V

Time : Three Hours] [Maximum Marks : 100

N.B.:— (1) Solve FIVE questions choosing ONE from each of the four units.

(2) Question No. 9 is compulsory.

UNIT—I

1. (a) Find the solution of the integro-differential equation $u'(x)+\int_0^1 e^{x-t}.u(t)dt=1,\ 0\le t\le 1$, where u (0) = 0 . 10

(b) Reduced voltera integral equation of first kind to a voltera integral equation of second kind. 10

2. (a) Write general form of linear integral equation and write classification of linear integral equation. 10

(b) Transform the problem $L_y = f(x),\ x_1 \le x \le x_2$ with boundary conditions $a_1 y(x_1)+b_1 y'(x_1)=0$, $a_2 y(x_2)+b_2 y'(x_2)=0$ into an integral equation. 10

UNIT—II

3. (a) The eigen functions associated with a Hermitian kernel form an orthonormal set. 10

(b) State and prove Hilbert - Schmidt Theorem. 10

4. (a) Find the fourier series solution for the integral equation

$$f(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1-\alpha^2}{1-2.\alpha\cos(x-y)+\alpha^2}.\phi(y).dy,\ 0<\alpha<1,\ -\pi\le x\le\pi.$$ 10

(b) Show that the kernel

$$k(x,y)=\sum_{n=1}^{\infty}\frac{\sin nx.\cos ny}{n+i},\ -\pi\le x,y\le\pi$$

does not have any eigen values and find the corresponding K_R and K_L. 10

UNIT—III

5. (a) Solve the integral equation $\int_0^x \sin\alpha(x-y)\phi(y)dy = x$. 10

(b) Solve the integral equation $\phi(x)=2\cos ax+\int_0^x (x-t).\phi(t)dt,\ x\ge 0$. 10

6. (a) Find the resolvent Kernel of the integral equation

$$\phi(x,y)=f(x,y)+\int_0^x\int_0^y e^{(x-\xi)+(y-\eta)}\cdot\phi(\xi,\eta)\,d\xi\,d\eta$$ and hence write the solution. 10

(b) Solve the integral equation $\phi''(x)=1-\int_0^x e^{2(x-t)}\phi'(t)\,dt$, where $\phi'(0)=0$ and $\phi(0)=0$. 10

UNIT—IV

7. (a) Find the first three functions in the sequence of functions arising from the iterative solution of the integral equation :

$$\phi(x)=x+\lambda\int_0^x\left[1+x(\phi(y))^2\right]dy.$$ 10

(b) Solve the integral equation $\phi(x)=\int_0^x\left[\frac{1+\phi(y)}{1+y}\right]dy$. 10

8. (a) Find the approximations to $\phi\left(\frac{1}{4}\right)$ and $\phi\left(\frac{3}{4}\right)$ when $\phi(x)$ is determined by the integral equation $\phi(x)-\int_0^1 e^{xy}\phi(y)\,dy=1-x^{-1}(e^x-1)$. 10

(b) Solve the integral equation $\frac{a}{a^2+r^2}=\int_0^\infty \cos\omega x.\phi(\omega)\,d\omega$, $a > 0$. 10

Compulsory Question

9. (a) Find the integral equation corresponding to $y''(x) + 2xy'(x) + y(x) = 0$, with $y(0) = 1$, $y'(0) = 0$. 5

(b) If the eigen values of Hermitian Kernel then show that they are real. 5

(c) Solve the integral equation $x^2=\int_0^x \sin a(x-y)\phi(y)\,dy$, $a \neq 0$. 5

(d) Solve $\int_0^\ell \frac{h(v)}{u-w}\,du=1,\ 0\le w\le \ell$. 5

PSM/KW/23/10003

Master of Science (M.Sc.) Mathematics Sem.—I (CBCS) New Education Policy (NEP) Examination

MMT1T03 M3 : ORDINARY DIFFERENTIAL EQUATIONS

Compulsory Paper—3

Time : Three Hours] [Maximum Marks : 80

Note :—Solve **FIVE** questions, choosing **ONE** from each Unit. Question No. **9** is compulsory.

UNIT—I

1. (a) Find the solution of the Legendre equation :

$$L(y) = (1 - x^2)y'' - 2xy' + \alpha(\alpha + 1) = 0.$$ 8

(b) One solution of $x^2y'' - xy' + y = 0$ on $0 < x < \infty$ is $\varphi_1(x) = x$. Find all solutions of $x^2y'' - xy' + y = x^2$. 8

OR

2. (a) Let ϕ be any solution of $L(y) = y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_0 then show that for all x in I :

$$\|\phi(x_0)\| e^{-k|x - x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{k|x - x_0|}$$

where $k = 1 + |a_1| + |a_2|$. 8

(b) Determine the series solution for differential equation :

$$y'' - xy' = 0 \text{ around } x_0 = 0.$$ 8

UNIT—II

3. (a) Solve the following differential equations :

(i) $x^2y'' - 5xy' + 6y = 0$

(ii) $2x^2y'' + xy' - y = 0$ 8

(b) Show that :

(i) $(xJ'_\alpha(x)) = \alpha J_\alpha(x) + xJ_{\alpha+1}(x)$

(ii) $\dfrac{d}{dx}(x^{-\alpha}J_\alpha(x)) = -x^{-\alpha}J_{\alpha+1}(x).$ 8

OR

4. (a) Derive the formula for the Bessel function of zero order of first kind 8

(b) Find all solution φ of the form $\varphi(x) = |x|^r \sum_{k=0}^{\infty} c_k x^k$ for the equation $x^2y'' + \frac{3}{2}xy' + xy = 0$. 8

UNIT—III

5. (a) Find the first three successive approximations for the equations :

(i) $y' = x^2 + y^2, \quad y(0) = 2$

(ii) $y' = y^2, \quad y(0) = 1.$ 8

(b) Explain the method of successive approximation in detail. 8

OR

6. (a) Define Lipschitz condition, further show that $f(x, y) = x^2 + y^2$ satisfy Lipschitz condition on the region $S : |x| \leq 1, |y| \leq 1.$ 8

(b) Find the solution of $y' = xy, \; y(0) = 1$ by successive approximation. 8

UNIT—IV

7. (a) Find the solution φ of the system :

$$y'_1 = y_1 + y_2$$

$$y'_2 = y_1 + y_2 + e^{3x} \text{ satisfying } \varphi(0) = (0, 0).$$ 8

(b) Define Orthogonality of eigenfunctions, further solve following Sturm-Liouville equation :

$$y'' + \lambda y = 0, \quad y(0) = 0, \; y(l) = 0.$$ 8

OR

8. (a) Solve the following :

(i) $y'' + e^x y' = e^x$

(ii) $y'' = yy'.$ 8

(b) Show that following vector valued function satisfies Lipschitz condition and compute the corresponding Lipschitz constant :

(i) $R : |x| \leq \infty, |y| \leq \infty \; f(x, y) = (3x + 2y_1, \; y_1 - y_2)$

(ii) $R : |x| \leq 1, |y| \leq 1 \; f(x, y) = (y_2^2 + 1, \; x + y_1^2).$ 8

(Compulsory Question)

9. (a) Find second independent solution of the differential equation $x^2y'' - 7xy' + 15y = 0.$ If $\varphi_1(x) = x^3$ is one of the solutions.

(b) Show that $x^{\frac{1}{2}} J_{\frac{1}{2}}(x) = \dfrac{\sqrt{2}}{\Gamma 1/2} \sin(x).$

(c) Prove that $f(x, y) = xy^2$ satisfy Lipschitz condition on the region $R : |x| \leq 1, |y| \leq 1$ but do not satisfy Lipschitz condition on the region $S : |x| \leq 1, |y| \leq \infty.$

(d) Solve : $y'' + y' = 1.$ $4 \times 4 = 16$