International Journal of Scientific Research in Physics and Applied Sciences **Research Paper**
Vol.10, Issue.6, pp.01-07, December (2022) **E-ISSN:** 2348-3423

Anisotropic String Cosmological Model for Perfect Fluid Distribution in *f(T)* Gravity

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Available online at: www.isroset.org | DOI: https://doi.org/10.26438/ijsrpas/v10i6.17

Received: 18/Oct/2022, Accepted: 21/Nov/2022, Online: 31/Dec/2022

***Abstract*—**The present study deals with a spatially homogeneous and anisotropic Bianchi type VI_0 space-time in the presence of string of clouds and perfect fluid distribution within the context of $f(T)$ gravity. The analytic relation $B = A^n$ and the hybrid exponential form of the average scale factor have been considered to obtain the exact solutions of highly non-linear field equations. Some physical and kinematical parameters of the constructed model along with the behavior of the equation of state parameter for string fluid are discussed and presented graphically.

***Keywords*—**Bianchi-type VI_0 space-time, Cosmic String, Perfect Fluid, $f(T)$ Gravity.

I. INTRODUCTION

The origin and evolution of the universe are one of the oldest problems of modern cosmology, which has no satisfactory explanations. The cosmic strings build appreciable interest as these are believed to play an important role during the early evolution and late-time accelerated expansion of the universe [1],[2],[3],[4],[5],[6]. One cannot predict the count of visible cosmic strings in the universe but their existence may play an important role [7],[8]. The cosmic strings hold stress energy and coupled gravitational field, which increases one's interest to study the gravitational effect which originates from strings. Letelier [9],[10], initiated the general relativistic operation of strings and investigated the solution of Einstein's field equations for a cloud of string with spherical, plane, and cylindrical symmetry along with the string cloud models. Stachel [11], developed the model for the thickening of string perfect dust, its generalization to null strings, and to perfect fluid of strings. Vilenkin [12] studied some observational effects of strings as well as some distinct features of the string scenario like galaxy formations, the baryon-dominated universe, the neutrino-dominated universe, and the axion-dominated universe, etc. Recently various string-coupled, as well as perfect fluid-coupled cosmological models [13],[14],[15],[16],[17],[18],[19],[20],[21],[22],[23],[24], have been studied by researchers which develop our interest in studying perfect fluid-coupled string cosmology.

In recent years, modified theories of gravity have received immense success to study the unknown and hidden aspects of the universe theoretically. Modified theories of gravity which are developed from Einstein's – Hilbert action principle are universally gaining more and more interest from researchers. The $f(T)$ theory of gravitation which is the generalization of the TEGR i.e. teleparallel equivalent of general relativity is a widely accepted theory for possible substitution that explains the early inflation and late-time cosmic accelerated expansion. Ferraro [25], concisely reviewed the $f(R)$, and $f(T)$ gravity theories and showed some remarkable applications to cosmology and cosmic strings. Böhmer et.al. [26], studied the entity of relativistic stars and specifically constructed classes of solutions for a static perfect fluid. Moreira et.al. [27], have examined solutions for $l = 0$ vertex which yield thick-string-like brane in $f(T)$ gravity.

Many researchers have constructed several types of string cosmological models for perfect fluid or pressureless distribution (dust distribution). Chirde et al. [28], examined perfect fluid coupled string cosmology using the Bianchi-type I metric and constructed the accelerating, expanding, and anisotropic cosmological model. Gaikwad et al. [29], using the Bianchi-type I metric analyzed the massive string magnetized barotropic perfect fluid cosmological model. A string of clouds in presence of perfect fluid and decaying vacuum energy density Λ have been analyzed by Pradhan et al., [30]. Bali & Pareek as well as Rani et al. [31],[32], have studied magnetized string cosmology for perfect fluid distribution using Bianchi type III space-time. Taking account of the above research work we have considered Bianchi Type VI_0 Space-time to construct the string cosmological model for perfect fluid distribution.

This paper is divided into several sections: Section II deals with elementary definitions and equations of motion in the framework of $f(T)$ gravity. In Section III considering spatially homogeneous and anisotropic Bianchi type VI_0 metric, we have obtained the corresponding field equations in the framework of $f(T)$ gravity. In Section IV, we obtained the exact solution of the field equations along with various physical and kinematical quantities and discussed their properties through graphical representation. Lastly, in Section V, we have concluded the investigations.

II. Methodology and Equation of Motion

In this section, we provide a concise explanation of $f(T)$ gravity and a thorough derivation of its field equations.
The line element for a general space-time is defined as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{1}$$

where $g_{\mu\nu}$ are the components of the metric tensor which are symmetric. The above line element can be transformed into the Minkowskian space-time (which represents the dynamic fields of the theory) as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j, \tag{2}$$

$$dx^\mu = e_i^\mu \theta^i, \quad \theta^i = e_\mu^i dx^\mu, \tag{3}$$

in which η_{ij} are metric tensors in Minkowskian space-time such that $\eta_{ij} = diag[1,-1,-1,-1]$ and $e_i^\mu e_\nu^i = \delta_\nu^\mu$ or $e_i^\mu e_\mu^j = \delta_i^j$. $\sqrt{-g} = \det[e_\mu^i] = e$ and the dynamic fields of the theory are represented by the tetrads matrix e_μ^α. The Weitzenbocks connection components which have a zero curvature but nonzero torsion for a manifold are defined as

$$\Gamma^\alpha{}_{\mu\nu} = e_i^\alpha \partial_\nu e_\mu^i = -e_\mu^i \partial_\nu e_i^\alpha. \tag{4}$$

The components of the torsion tensor for a manifold are defined by the anti-symmetric part of the Weitzenbocks connection

$$T^\alpha{}_{\mu\nu} = \Gamma^\alpha{}_{\mu\nu} - \Gamma^\alpha{}_{\nu\mu} = e_i^\alpha(\partial_\mu e_\nu^i - \partial_\nu e_\mu^i). \tag{5}$$

The con-torsion tensor components are defined by

$$K^{\mu\nu}{}_\alpha = -\frac{1}{2}\left(T^{\mu\nu}{}_\alpha - T^{\nu\mu}{}_\alpha - T_\alpha^{\mu\nu}\right). \tag{6}$$

A new tensor, $S_\alpha{}^{\mu\nu}$ constructed from the components of the torsion and con-torsion tensors for a better understanding of the definition of the scalar equivalent to the curvature scalar of Riemannian geometry as follows,

$$S_\alpha{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_\alpha + \delta_\alpha^\mu T^{\beta\nu}{}_\beta - \delta_\alpha^\nu T^{\beta\mu}{}_\beta\right). \tag{7}$$

The torsion scalar is defined using the contraction which is similar to the scalar curvature in GR as

$$T = T^\alpha{}_{\mu\nu} S_\alpha{}^{\mu\nu}. \tag{8}$$

The action is defined by generalizing the teleparallel gravity, i.e, $f(T)$ theory as

$$S = \int [f(T) + L_{Matter}] e d^4x, \tag{9}$$

where $f(T)$ denotes an algebraic function of the torsion scalar T.
Equations of motion are obtained by functional variation of the action (9) with respect to the tetrads as

$$S_\mu{}^{\nu\rho}\left(\partial_\rho T\right) f_{TT} + \left\{e^{-1} e_\mu^i \partial_\rho \left(e e_i^\alpha S_\alpha{}^{\nu\rho}\right) + T^\alpha{}_{\lambda\mu} S_\alpha{}^{\nu\lambda}\right\} f_T + \frac{1}{4}\delta_\mu^\nu f = 4\pi T_\mu^\nu, \tag{10}$$

where T_μ^ν is the energy-momentum tensor for string of clouds with perfect fluid distribution, f_T and f_{TT} denotes respectively the first and second order derivatives of $f(T)$ with respect to T. For $f(T)=$ constant, the equations of motion in (10) reduce to the equations of motion of the teleparallel gravity with a cosmological constant, which is dynamically equivalent to general relativity. These equations depend on the choice made for the set of tetrads.

III. Metric and Field Equations

We consider a spatially homogeneous and anisotropic Bianchi type-VI$_0$ line element

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2x}dy^2 - C^2(t)e^{2x}dz^2, \tag{11}$$

where the scale factors $A, B,$ and C are functions of cosmic time t only.
Consider the set of diagonal tetrads related to the metric (11) as

$$[e_\mu^\nu] = diag[1, A, Be^{-x}, Ce^{x}]. \tag{12}$$

Then the determinant of the matrix (11) is

$$e = ABC. \tag{13}$$

The torsion scalar from (8) is obtained as

$$T = -2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - 1\right). \tag{14}$$

Considering the energy-momentum tensor for string cloud with perfect fluid distribution as

$$T_\mu^\nu = (\rho + p)u_\mu u^\nu - p g_\mu^\nu - \lambda x_\mu x^\nu, \tag{15}$$

in which u^μ denotes a four-velocity vector and x^μ denotes a unit space-like vector of the cloud string satisfying the conditions, $u^\mu u_\mu = 1 = -x^\mu x_\mu$ and $u^\mu x_\nu = 0$, for $\mu \neq \nu$ and ρ is the proper energy density of the particle, p is the isotropic pressure, λ is the strings tension density.
In a co-moving coordinate system, we have

$$u^\mu = (0,0,0,1), \quad x^\mu = \left(A^{-1},0,0,0\right), \tag{16}$$

If the configuration of particle density is indicated by ρ_p, then we assume

$$\rho = \rho_p + \lambda. \tag{17}$$

We obtained the field equations for Bianchi type-VI$_0$ space-time (11), from (10) in the framework of $f(T)$ gravity as

$$f + 4f_T\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right) = 16\pi\rho, \tag{18}$$

$$f+2f_T\left(\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}-2\right)+2\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}$$
$$=16\pi(\lambda-p), \quad (19)$$

$$f+2f_T\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+2\frac{\dot{A}\dot{C}}{AC}+\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}$$
$$=-16\pi p, \quad (20)$$

$$f+2f_T\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=-16\pi p, \quad (21)$$

$$\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)f_T=0, , \quad (22)$$

where the overhead dot (.)denotes the derivative with respect to cosmic time t. Here we have five highly non-linear differential field equations with seven unknowns, namely; $f, A, B, C, p, \lambda,$ and ρ . Since $f(T)$ is non-vanishing in TEGR, from (22) one can get $B = k\,C$ which yields $B = C$ for $k = 1$. Then we obtain

$$f+4f_T\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)=16\pi\rho, \quad (23)$$

$$f+4f_T\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}-1\right)+4\left(\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=16\pi(\lambda-p), \quad (24)$$

$$f+2f_T\left(3\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=-16\pi p. \quad (25)$$

Finally, we have three non-linear differential equations with six unknowns, namely $f, A, B, p, \lambda, \rho$. The solution to these equations is discussed in the next section.

IV. SOLUTIONS OF FIELD EQUATIONS

To solve the field equations completely to obtain the exact solutions, we consider the linear $f(T) = T$ gravity and assume that the expansion scalar (θ) is proportional to the shear scalar (σ^2), which leads to an analytic relation as

$$B = A^n. \quad (26)$$

We find some kinematical space-time quantities, as follows:

The average scale factor (a) and the spatial volume (V) are respectively defined as

$$a=\sqrt[3]{B^{n+2}}, \quad V=a^3. \quad (27)$$

Also, the volumetric expansion rate of the universe is described by the generalized mean Hubble's parameter (H) given by

$$H=\frac{1}{3}\sum_{i=1}^{3}H_i=\frac{1}{3}(H_1+H_2+H_3), \quad (28)$$

in which H_1, H_2, and H_3 denotes the directional Hubble parameters.

From Eqns. (27) and (28), we get

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\sum_{i=1}^{3}H_i=\frac{\dot{a}}{a}. \quad (29)$$

To analyze, whether the model approaches isotropy or not, we discuss the mean anisotropy parameter (A_m) as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H}-1\right)^2. \quad (30)$$

The expansion scalar (θ) and the shear scalar (σ^2) are respectively defined as

$$\theta=u^{\mu}_{;\mu}=3H, \quad (31)$$

$$\sigma^2=\frac{3}{2}A_m H^2, \quad (32)$$

The deceleration parameter is defined as

$$q=-\frac{\ddot{a}a}{\dot{a}^2}=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \quad (33)$$

We have considered the hybrid exponential form of an average scale factor as

$$a=me^{\alpha t}t^{\beta}. \quad (34)$$

Then from (26), (27), and (34), we obtained the metric coefficients A , and B as

$$A=\left(me^{\alpha t}t^{\beta}\right)^{\frac{3}{(2n+1)}}, \text{ and } B=C=\left(me^{\alpha t}t^{\beta}\right)^{\frac{3n}{(2n+1)}}. \quad (35)$$

Substituting A and B from (35) in (11), we get

$$ds^2=dt^2-\left(me^{\alpha t}t^{\beta}\right)^{\frac{6}{(2n+1)}}dx^2-\left(me^{\alpha t}t^{\beta}\right)^{\frac{6n}{(2n+1)}}e^{-2x}dy^2$$
$$-\left(me^{\alpha t}t^{\beta}\right)^{\frac{6n}{(2n+1)}}e^{2x}dz^2. \quad (36)$$

From (14) we have obtained the Torsion scalar as

$$T=\frac{2(2n+1)^2t^2-18n(n+2)(\alpha t+\beta)^2}{(2n+1)^2t^2}. \quad (37)$$

In the following, we have determined the spatial volume (V) , the mean Hubble's parameter (H) , the expansion scalar (θ), the mean anisotropy parameter (A_m), and the shear scalar (σ^2) respectively as.

$$V=\left(me^{\alpha t}t^{\beta}\right)^3, \quad (38)$$

$$H=\frac{\alpha t+\beta}{t}, \quad (39)$$

$$\theta = \frac{3(\alpha t + \beta)}{t}, \tag{40}$$

$$A_m = \frac{2(n-1)^2}{(n+2)^2}, \tag{41}$$

$$\sigma^2 = \frac{3(n-1)^2(\alpha t + \beta)^2}{(n+2)^2 t^2}, \tag{42}$$

we observe from Fig.1 that at an initial epoch when $t = 0$ the spatial volume is null. Thus initially the constructed model is singularity free whereas with increasing time the volume of the model starts increasing exponentially for $t > 2.467$ which shows the cosmic expansion in later times. Fig.2 shows that the mean Hubble's parameter, the expansion scalar, and the shear scalar were too large in the early era but diminished instantly for the range $0 < t < 1$ and becomes finitely stable with time elevation. The mean anisotropy parameter is constant throughout the expansion. Also, the ratio $\sigma^2/\theta^2 \neq 0$ shows that the constructed model doesn't approach isotropy.

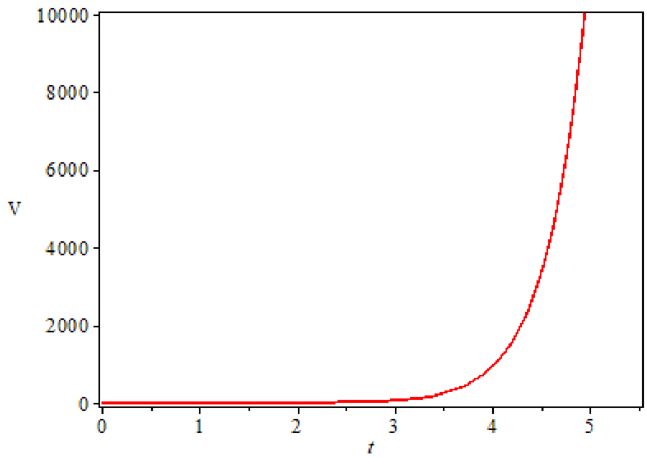

Figure 1: Variation of V Vs. t
for $\alpha = 0.6$, $\beta = 1.075$ and $m = 0.2$

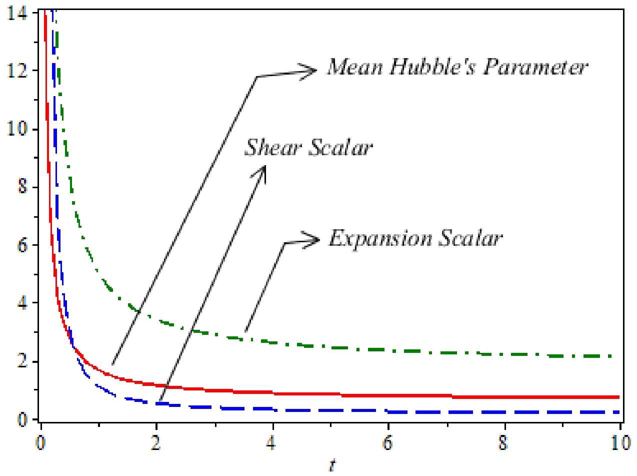

Figure 2: Variation of H, θ, and σ^2 Vs. t
for $\alpha = 0.6$, $\beta = 1.075$ and $n = 0.2$

The deceleration parameter

$$q = -1 + \frac{\beta}{(\alpha t + \beta)^2}. \tag{43}$$

As depicted in Fig.3 we observe that the deceleration parameter diminishes monotonically with an increase in cosmic time and later it approaches -1 when $t \to \infty$, showing the accelerated cosmic evolution which is in good agreement with recent observational data.

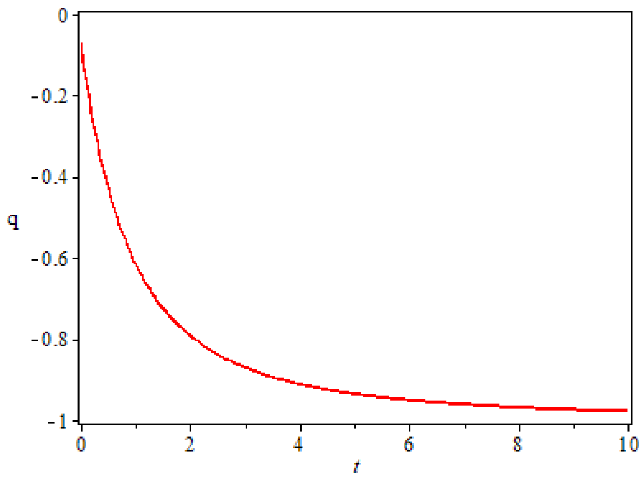

Figure 3: Variation of q Vs. t
for $\alpha = 0.6$, and $\beta = 1.075$

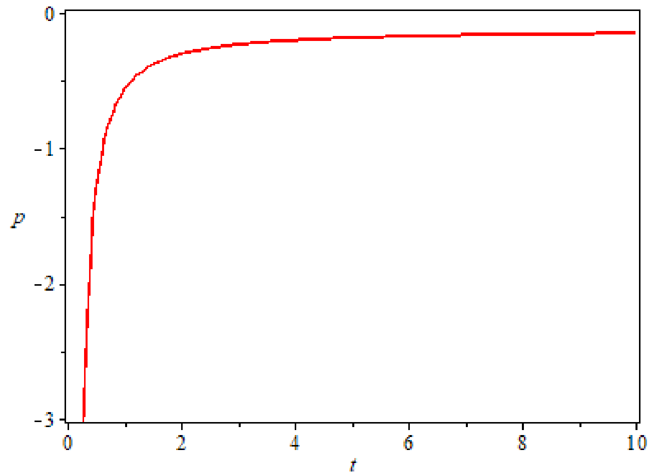

Figure 4: Variation of p Vs. t
for $\alpha = 0.6$, $\beta = 1.075$ and $n = 0.2$

From (25), we have obtained the pressure as

$$p = -\frac{\left\{ \begin{array}{l} (n^2+n+1)\left[(9\alpha^2+4)t^2 + 18\alpha\beta t + 9\beta^2\right] \\ -3(2n^2+3n+1)\beta - 3t^2 \end{array} \right\}}{8\pi(2n+1)^2 t^2}. \tag{44}$$

From Fig.4 we noticed that the pressure expands in negative with an increase in cosmic time and approaches a small constant value -0.1213.

Using (44) in (24), we have obtained tension density as

$$\lambda = \frac{\left\{9\alpha^2(n-1)-2(2n+1)\right\}t^2+3\beta(n-1)\left\{3(2\alpha t+\beta)-1\right\}}{8\pi(2n+1)t^2}. \tag{45}$$

From (23), we have obtained the value of energy density as

$$\rho = \frac{\left\{\begin{array}{l}\left[\left(9\alpha^2+4\right)t^2+18\alpha\beta t+9\beta^2\right]n^2\\ +\left[\left(18\alpha^2+4\right)t^2+36\alpha\beta t+18\beta^2\right]n+t^2\end{array}\right\}}{8\pi(2n+1)^2t^2}, \tag{46}$$

Using (45) & (46) in (17) we have obtained particle density as

$$\rho_p = -\frac{3\left\{\begin{array}{l}\left[\left(3\alpha^2-4\right)t^2+6\alpha\beta t+3\beta^2-2\beta\right]n^2\\ -\left[\left(9\alpha^2+4\right)t^2+18\alpha\beta t+9\beta^2-\beta\right]n\\ -\left(3\alpha^2+1\right)t^2-6\alpha\beta t-3\beta^2+\beta\end{array}\right\}}{8\pi(2n+1)t^2}. \tag{47}$$

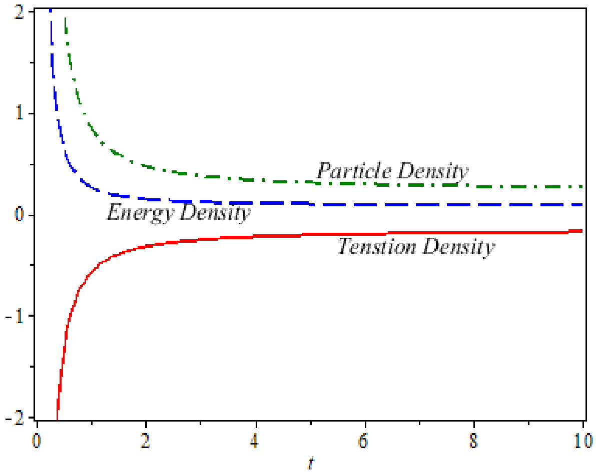

Figure 5: Variation of ρ, ρ_p, and λ Vs. t for $\alpha = 0.6$, $\beta = 1.075$ and $n = 0.2$

It is observed from Fig.5 that in the early era particle density and energy density diminish from infinite positive to small constant values $\rho = 0.069$, and $\rho_p = 0.254$ with an increase in cosmic time, while tension density evolves in negative with an increase in cosmic time to approach a constant ultimately. The comparative behavior $\rho_p/|\lambda|$ is depicted in Fig.6. Letelier [33], presented the Satchel string of cloud model and mentioned that if $\lambda < 0$ the string phase disappears. The comparative physical behavior of ρ_p and λ is not only positive throughout the evolution of the universe but also $\rho_p/|\lambda| > 1$ implies that $\rho_p > \lambda$, attributing the particle-dominated phase which is supported by [34],[35],[36].

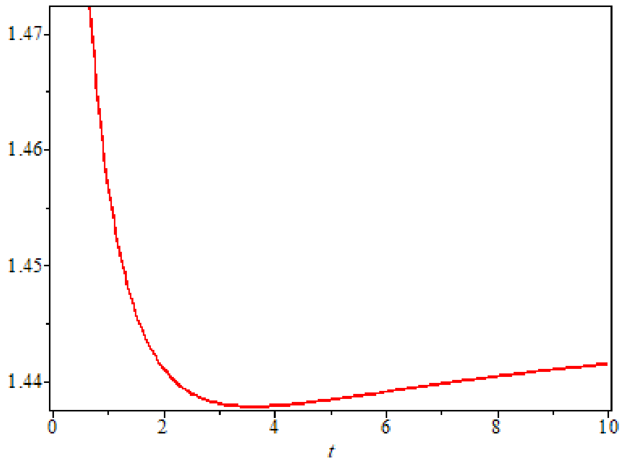

Figure 6: Representation of comparative behavior $\rho_p/|\lambda|$ Vs. t

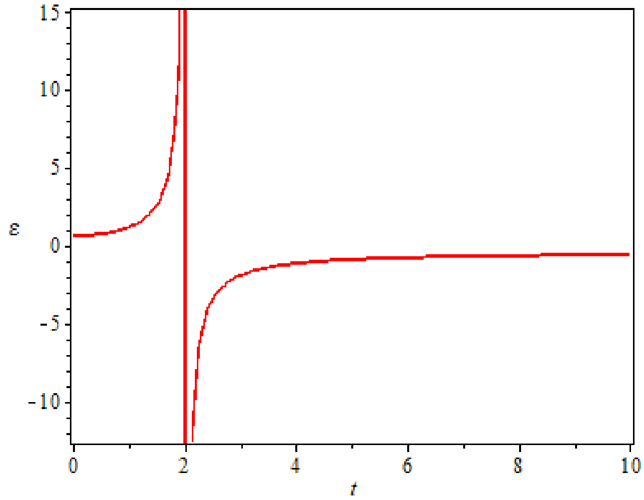

Figure 7: Variation of ε Vs. t for $\alpha = 0.6$, $\beta = 1.075$ and $n = 0.2$

Now, the equation of state (EoS) for string fluid is given by

$$\rho = \varepsilon\lambda. \tag{48}$$

From (45) and (46) in (48) we have obtained the EoS parameter for the string fluid as

$$\varepsilon = \frac{\left\{\begin{array}{l}\left[\left(9\alpha^2+4\right)t^2+18\alpha\beta t+9\beta^2\right]n^2\\ +\left[\left(18\alpha^2+4\right)t^2+36\alpha\beta t+18\beta^2\right]n+t^2\end{array}\right\}}{\left\{\begin{array}{l}\left[9\alpha^2\left(2n^2-n-1\right)-2(2n+1)^2\right]t^2\\ +3\beta\left(2n^2-n-1\right)\left[3(2\alpha t+\beta)-1\right]\end{array}\right\}}. \tag{49}$$

Fig.7 shows that the EoS parameter increases exponentially for $0 < t < 1.87$ and falls (forming a cluster) in the small neighborhood of $t = 2$ and again rises to approach a constant negative value -0.44.

V. CONCLUDING

In this paper, we have obtained the exact solutions to field equations of Bianchi type VI_0 metric in the presence of a string of clouds coupled with perfect fluid distribution within the context of $f(T)$ gravity. In our investigations, we observed that from $t > 2.467$ the spatial volume of the universe increases exponentially which shows cosmic expansion in later times but the decrease in mean Hubble's parameter, expansion scalar, and shear scalar with an increase in cosmic time alongwith the mean anisotropy parameter confirms that the rate of expansion is decreased and model do not approach isotropy. Also from Fig.3, it is observed that the universe is in an accelerating phase, thus our constructed model behaves like a present universe which is anisotropic, accelerating, and expanding. It is observed that particle density and energy density are always positive and are the decreasing functions of cosmic time, while the tension density is always negative which demonstrates the string phase disappears which is supported by [33]. Also, the comparative physical behavior attributing the particle-dominated phase which is in good agreement with [34],[35],[36]. We have observed that the pressure for this model is always negative and the EoS parameter of the cosmic string is initially positive, i.e. it gives the matter-dominated universe and later times becomes negative which is in good agreement with the recent observational data.

ACKNOWLEDGMENT

The authors would like to place on record their sincere thanks to the Editor and anonymous Referees for their valuable comments which have helped in improving the quality of the paper.

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