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# Bulk Viscous String Cosmological Model with Constant Deceleration Parameter in Teleparallel Gravity

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**Abstract**— In this article, we have studied the Bianchi-type V cosmological models which are spatially homogeneous and anisotropic in presence of bulk viscous fluid coupled with one-dimensional cosmic string having the constant deceleration parameter. We have obtained the exact solutions of highly non-linear field equations considering linear $f(T)=T$ gravity, the equation of state, and the spatial law of variation for Hubble's parameter. Some physical and kinematical properties of the constructed models have been discussed and presented graphically and it is interesting to note that the resultant models are in good agreement with recent observations.

***Keywords***—Bianchi-type V space-time, Bulk Viscous Fluid, Cosmic String, Teleparallel Gravity.

## I. INTRODUCTION

Measurements from high redshift supernovae observed by researchers provide direct evidences for an accelerating universe [1],[2],[3],[4],[5],[6],[7],[8]. The cause of the acceleration of the universe is unknown and is commonly considered as a dark energy problem which is due to the negative pressure of the universe. To report this issue theoretically two different approaches are there, one is to develop viable dark energy models and another is the modification of Einstein's general theory of relativity, in which it has been observed that Einstein's field equations have been modified to explain the current phase of the universe as well as many hidden aspects of modern cosmology like black holes, warm holes, etc. Many researchers have constructed several types of theoretical cosmological models to study these features in modified gravity theories. Amongst them, $f(T)$ theory of gravitation is a viable candidate which is based on the modification of the teleparallel equivalent of general relativity. $f(T)$ gravity theory was initially used to study inflation without an inflaton [9], then cosmic acceleration has been investigated with the absence of dark energy [10], and avoiding pathologies [11]. Cosmological perturbations have also been studied in $f(T)$ gravity [12],[13],[14], and several researchers [15],[16],[17],[18], have performed the reconstruction of $f(T)$ gravity.

Bulk viscosity performs a significant role in cosmology and presents cosmic accelerated expansion popularly known as the inflationary phase. The theoretical development of the universe and the effects of bulk viscosity on cosmic evolution have been examined by numerous cosmologists and physicists using the source as a bulk viscous fluid. Santhi et al., [19] studied the accelerated expansion in the context of $f(R)$ gravity using power and exponential law models. Considering the Kantowski-Sachs metric Reddy et al., [20] have constructed an isotropic bulk viscous string cosmological model showing the special case for the non-validating cosmic strings. Hegazy, [21] developed the formula for calculating cosmic entropy in terms of viscosity and try it to examine the entropy, enthalpy, Gibbs energy, and Helmholtz energy of the constructed model in presence of viscosity. Naidu et al., [22],[23],[24],[25],[26],[27], vigorously studied string cosmological models in reference to different theories of gravitation. Some recent and important investigations of bulk viscous fluid in presence of cloud strings have been obtained by several cosmologists [28],[29],[30],[31],[32],[33],[34],[35] in different contexts.

Strings are widely studied by researchers as they play an important role in explaining the early phase of cosmic evolution. Nojiri et al. [36] studied string-inspired models, inflation, bounce, and late-time evolution in reference to modified gravity. Freidel et al., [37] discussed the formulation and dynamics of string theory and looks for string solutions. Mishra et al. [38] investigated the string cosmological model using spatially homogeneous and anisotropic Bianchi type V space-time. The viscous string cosmological models explain the cosmic accelerated expansion and similar work has been done by Vinutha et al., [39]. Darabi et al., [40] obtained string cosmological solutions via Hojman symmetry using FRW line element. Chirde et al., [41] have studied the LRS Bianchi type I cosmological model having the source as perfect fluid and a string of clouds using three different $f(T)$ formalisms. The substantial theoretical development of string theory as well as a bulk viscous fluid which has motivated our perspective and modified our investigations [42],[43],[44],

 

[45],[46],[47],[48],[49],[50],[51],[52],[53],[54],[55],[56] has been done using different theories of gravitation.

Many researchers have constructed various bulk viscous and string cosmological models as given in the above literature explaining inflation, evolution, accelerated expansion, and many hidden aspects of the universe which has encouraged our motive and led to this work. Inspired by the above literature review and studies we have considered spatially homogeneous and anisotropic Bianchi type V space-time to construct the bulk viscous coupled string cosmological model within the context of $f(T)$ gravity.

This paper is divided into several sections: Section II deals with the metric, an equation of motion, and its field equations within the framework of $f(T)$ gravity. In section III, we have obtained the exact solution of highly non-linear differential field equations along with different physical and kinematical quantities. Lastly, in section IV, we have concluded the investigations.

## II. Metric, Methodology, and Field Equations

We consider a line element

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left(B^2(t)dy^2 + C^2(t)dz^2\right), \quad (1)$$

which is a spatially homogeneous and anisotropic Bianchi type-V metric in which $m$ is constant and $A, B, C$ are functions of cosmic time $t$ only.

In teleparallel gravity the tetrad is given by $g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu$, where $\mu$ and $\nu$ are coordinate indices. For a given metric $\sqrt{-g} = \det[e^i_\mu] = e$ and the dynamic field theory are represented by the infinite tetrad fields $e^\alpha_\mu$ which satisfy $e^\mu_i e^i_\nu = \delta^\mu_\nu$ or $e^\mu_i e^j_\mu = \delta^j_i$. Now the set of diagonal tetrads related to the metric (1) is as

$$[e^\nu_\mu] = diag[1, A, Be^{mx}, Ce^{mx}]. \quad (2)$$

The determinant of the matrix (1) is

$$e = ABCe^{2mx}. \quad (3)$$

Now we consider the action of generalizing the teleparallel gravity, i.e, $f(T)$ theory as

$$S = \int [T + f(T) + L_{Matter}] e d^4x, \quad (4)$$

where $f(T)$ denotes an algebraic function of the torsion scalar $T$. And the torsion scalar is defined using the contraction which is similar to the scalar curvature in general relativity as

$$T = T^\alpha{}_{\mu\nu} S_\alpha{}^{\mu\nu}. \quad (5)$$

where the tensors $T^\alpha{}_{\mu\nu}$, and $S_\alpha{}^{\mu\nu}$ are respectively as follows

$$T^\alpha{}_{\mu\nu} = \Gamma^\alpha{}_{\mu\nu} - \Gamma^\alpha{}_{\nu\mu} = e^\alpha_i(\partial_\mu e^i_\nu - \partial_\nu e^i_\mu), \quad (6)$$

$$S_\alpha{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_\alpha + \delta^\mu_\alpha T^{\beta\nu}{}_\beta - \delta^\nu_\alpha T^{\beta\mu}{}_\beta\right), \quad (7)$$

in which the Weitzenbocks connection components for a manifold, where the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) having only the existence of non-zero torsion terms, and con-torsion tensor components are respectively defined as

$$\Gamma^\alpha{}_{\mu\nu} = e^\alpha_i \partial_\nu e^i_\mu = -e^i_\mu \partial_\nu e^\alpha_i, \quad (8)$$

$$K^{\mu\nu}{}_\alpha = -\frac{1}{2}\left(T^{\mu\nu}{}_\alpha - T^{\nu\mu}{}_\alpha - T_\alpha{}^{\mu\nu}\right). \quad (9)$$

So from (5), we have obtained the torsion scalar as

$$T = -2\left\{\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + m^2\right\}. \quad (10)$$

Now the equations of motion are obtained by functional variation of the action (4) with respect to the tetrads as

$$S_\mu{}^{\nu\rho}\left(\partial_\rho T\right) f_{TT} + \left\{e^{-1} e^i_\mu \partial_\rho\left(e e^\alpha_i S_\alpha{}^{\nu\rho}\right) + T^\alpha{}_{\lambda\mu} S_\alpha{}^{\nu\lambda}\right\}(1 + f_T)$$
$$+ \frac{1}{4}\delta^\nu_\mu (T + f) = 4\pi T^\nu_\mu, \quad (11)$$

where the energy-momentum tensor $T^\nu_\mu$ is considered as bulk viscous fluid coupled with one-dimensional cosmic string, $f_T$ and $f_{TT}$ denotes respectively the first and second order derivatives of $f(T)$ with respect to $T$. For $f(T) =$ constant, the equations of motion in (11) reduce to the equations of motion of the teleparallel gravity with a cosmological constant, which is dynamically equivalent to general relativity. These equations depend on the choice made for the set of tetrads.

We consider the source as bulk viscous fluid coupled with a one-dimensional cosmic string given by

$$T^\nu_\mu = (\rho + \bar{p}) u_\mu u^\nu - \bar{p} g^\nu_\mu - \lambda x_\mu x^\nu, \quad (12)$$

And $$\bar{p} = p - 3\xi H, \quad (13)$$

where $\rho = \rho_p + \lambda$ is the proper string energy density with particles attached to them and $\rho_p$ is the particle energy density, $\lambda$ is the strings tension density, $3\xi H$ is bulk viscous pressure, $\xi(t)$ is the coefficient of bulk viscosity, $H$ is Hubble's parameter, $x^\nu$ denotes a unit space-like vector for the cloud string and $u^\nu$ denotes four-velocity vector satisfying the conditions, $u^\nu u_\nu = 1 = -x^\nu x_\nu$ and $u_\nu x^\nu = 0$.

In a co-moving coordinate system, we have

$$u^\nu = (0,0,0,1), \quad x^\nu = \left(A^{-1},0,0,0\right). \quad (14)$$

We obtained the field equations for Bianchi type-V space-time (1), from (11)-(14) in the framework of teleparallel gravity as

$$(T + f) + 2(1 + f_T)\left(\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + 2m^2\right) + 2\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{T} f_{TT}$$
$$= 16\pi\left(p - 3\xi H - \lambda\right), \quad (15)$$

$$(T+f)+2(1+f_T)\left(\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+2\frac{\dot{A}\dot{C}}{AC}+2m^2\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}$$
$$=16\pi(p-3\xi H), \tag{16}$$

$$(T+f)+2(1+f_T)\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+2m^2\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}f_{TT}$$
$$=16\pi(p-3\xi H), \tag{17}$$

$$(T+f)+4(1+f_T)\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right)=-16\pi\rho, \tag{18}$$

$$\left(2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)(1+f_T)=0, \tag{19}$$

$$\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)(1+f_T)=0, \tag{20}$$

where the overhead dot (□) denotes the derivative with respect to cosmic time $t$. By solving (19) and (20), one can get $A=D_0$, where $D_0$ is an integrating constant. Then (15)-(18) reduces to

$$(T+f)+2(1+f_T)\left(\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+2\frac{\dot{B}\dot{C}}{BC}+2m^2\right)+2\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}$$
$$=16\pi(p-3\xi H-\lambda), \tag{21}$$

$$(T+f)+2(1+f_T)\left(\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+2m^2\right)+2\left(\frac{\dot{C}}{C}\right)\dot{T}f_{TT}=16\pi(p-3\xi H), \tag{22}$$

$$(T+f)+2(1+f_T)\left(\frac{\ddot{B}}{B}+\frac{\dot{B}\dot{C}}{BC}+2m^2\right)+2\left(\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=16\pi(p-3\xi H), \tag{23}$$

$$(T+f)+4(1+f_T)\left(\frac{\dot{B}\dot{C}}{BC}\right)=-16\pi\rho. \tag{24}$$

Thus, we have four non-linear differential field equations with seven unknowns, namely $f,\ B,\ C,\ p,\ \xi,\ \rho,$ and $\lambda$; solutions which are discussed in the next section.

## III. SOLUTIONS OF FIELD EQUATIONS

To obtain the exact solutions of highly non-linear differential field equations (21)–(24) we have considered the following physically plausible conditions.

(i) we consider the linear form of $f(T)$ gravity as
$$f(T)=T. \tag{25}$$

(ii) As discussed by Reddy et al., [57] the combined effect of bulk viscous pressure and proper pressure is expressed as
$$\bar{p}=p-3\xi H=\varepsilon\rho, \tag{26}$$
where, $\varepsilon=\varepsilon_0-\beta \quad (0\le\varepsilon_0\le1), \quad p=\varepsilon_0\rho,$ (27)
and $\varepsilon_0$, and $\beta$ are constants.

(iii) We consider the special law of variation for Hubble's parameter presented by Berman [58].
$$H=\beta a^{-\gamma}, \tag{28}$$
where $\beta,$ and $\gamma$ are non-negative constants.

We find some kinematical space-time quantities, as follows:
The average scale factor $(a)$ and the spatial volume $(V)$ respectively as
$$a=\sqrt[3]{D_0BC}, \qquad V=a^3. \tag{29}$$
where $D_0$, is an integrating constant.

Also, the volumetric expansion rate of the universe is described by the generalized mean Hubble's parameter $(H)$ given by
$$H=\frac{1}{3}\sum_{i=1}^{3}H_i=\frac{1}{3}(H_1+H_2+H_3), \tag{30}$$
where $H_1=\frac{\dot{A}}{A},\ H_2=\frac{\dot{B}}{B},$ and $H_3=\frac{\dot{C}}{C}$ denotes the directional Hubble's parameters.
From Eqns. (29) and (30), we get
$$H=\frac{\dot{a}}{a}=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right). \tag{31}$$
To analyze, whether the model approaches isotropy or not, we discuss the mean anisotropy parameter $(A_m)$ as
$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H}-1\right)^2. \tag{32}$$
The expansion scalar $(\theta)$ and the shear scalar $(\sigma^2)$ are respectively defined as
$$\theta=u^{\mu}_{;\mu}=3H, \tag{33}$$
$$\sigma^2=\frac{3}{2}A_mH^2. \tag{34}$$
The deceleration parameter is defined as
$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{35}$$
From (28) and (31) we obtain
$$\dot{a}=\beta a^{-\gamma+1}, \tag{36}$$
$$\ddot{a}=-\beta^2(\gamma-1)a^{-2\gamma+1}. \tag{37}$$
From (36), (37) into (35), we get the useful constants in terms of the deceleration parameter $(q)$ as
$$q=\gamma-1,\ \text{for}\ \gamma\ne0, \tag{38}$$
$$q=-1,\quad \text{for}\ \gamma=0. \tag{39}$$
The sign of $q$ demonstrates whether the model is accelerating or not. The positive sign of the deceleration parameter $q$ i.e. for $\gamma>1$ corresponds to the decelerating phase of the universe although the deceleration parameter in $-1\le q<0$ corresponds to acceleration and for $q=0$ i.e.

 

for $\gamma=1$ corresponds to the evolution with a constant rate. And the observational evidences [1],[2],[3],[4],[5],[6],[7],[8] supports the accelerating phase of the universe.

So we have obtained the form for the average scale factor $(a)$ as

$$a=\alpha e^{\beta t}, \quad \text{for } \gamma=0, \tag{40}$$

And

$$a=(Dt+\eta)^{\frac{1}{\gamma}}, \quad \text{for } \gamma\neq 0. \tag{41}$$

where D is constant.

**CASE I: for $\gamma=0$, $(q=-1)$**

We have obtained the metric coefficients $A, B,$ and $C$ as

$$A=D_0,\ B=\frac{\alpha^3 e^{3\beta t}}{D_3 e^{\frac{9\alpha^3\beta^2 t+D_1 e^{-3\beta t}}{6\alpha^3\beta}}},\ \text{and } C=D_2 e^{\frac{9\alpha^3\beta^2 t+D_1 e^{-3\beta t}}{6\alpha^3\beta}} \tag{42}$$

where $D_1,\ D_2,$ and $D_3$ are constants.

From (42) into (1), we get

$$ds^2=dt^2-D_0^2dx^2-e^{2mx}\left\{\frac{\alpha^6 e^{6\beta t}}{D_3^2 e^{\frac{9\alpha^3\beta^2 t+D_1 e^{-3\beta t}}{3\alpha^3\beta}}}dy^2+D_2^2 e^{\frac{9\alpha^3\beta^2 t+D_1 e^{-3\beta t}}{3\alpha^3\beta}}dz^2\right\}. \tag{43}$$

From (10) we have obtained the torsion scalar as

$$T=-\frac{4m^2\alpha^6+9\alpha^6\beta^2-D_1^2e^{-6\beta t}}{2\alpha^6}. \tag{44}$$

Also, we have determined the volume $(V)$, the mean Hubble's parameter $(H)$, the expansion scalar $(\theta)$, the mean anisotropy parameter $(A_m)$, and the shear scalar $(\sigma^2)$ respectively as

$$V=\alpha^3 e^{3\beta t}, \tag{45}$$

$$H=\beta, \tag{46}$$

$$\theta=3\beta, \tag{47}$$

$$A_m=\frac{3\alpha^6\beta^2+D_1^2e^{-6\beta t}}{6\alpha^6\beta^2}, \tag{48}$$

$$\sigma^2=\frac{3\alpha^6\beta^2+D_1^2e^{-6\beta t}}{4\alpha^6}, \tag{49}$$

The graphical behavior of the volume $(V)$, the mean Hubble's parameter $(H)$, the expansion scalar $(\theta)$, the mean anisotropy parameter $(A_m)$, and the shear scalar $(\sigma^2)$ versus cosmic time $t$ has been depicted in Fig.1. It can be seen from Fig.1 that at an initial epoch when $t=0$ the volume $(V)$ is constant and starts increasing exponentially with an increase in cosmic time and diverges when $t\to\infty$, while the mean anisotropy parameter $(A_m)$ and the shear scalar $(\sigma^2)$ has a large value initially but decreases as cosmic time increases whereas the mean Hubble's parameter and the expansion scalar remains constant throughout the evolution. Hence the rate of cosmic expansion is constant throughout the evolution. Also, the ratio $\sigma^2/\theta^2\neq 0$ shows that the constructed model doesn't approach isotropy.

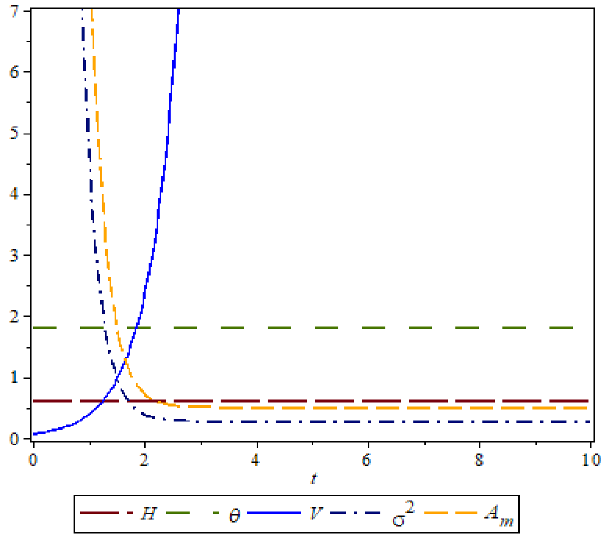


Figure 1: Variation of $V$, $H$, $\theta$, $A_m$, and $\sigma^2$ Vs. $t$ for $\alpha=0.4$, $\beta=0.6$, $m=2.3$, and $D_1=1.62$.

From (24), we have obtained the value of energy density as

$$\rho=\frac{\alpha^6\left\{4m^2-9\beta^2\right\}+D_1^2e^{-6\beta t}}{16\pi\alpha^6}. \tag{50}$$

From (21), and (22) we have obtained the value of tension density as

$$\lambda=-\frac{9\beta^2}{8\pi}. \tag{51}$$

Also, we have obtained the particle density as

$$\rho_p=\frac{\alpha^6\left\{4m^2+9\beta^2\right\}+D_1^2e^{-6\beta t}}{16\pi\alpha^6}. \tag{52}$$

The physical behavior of energy density $(\rho)$, tension density $(\lambda)$, particle density $(\rho_p)$, and the comparative behavior $\rho_p/|\lambda|$ is depicted in Fig.2. At an initial epoch, energy density $(\rho)$ and particle density $(\rho_p)$ decreases from positive, and eventually both approach a constant value while the tension density $(\lambda)$ is constant in negative throughout the evolution. [59], presented the Satchel string of cloud model and mentioned that if $\lambda<0$ the string phase disappears. The comparative behavior $\rho_p/|\lambda|>1$ implies that $\rho_p>|\lambda|$, attributing the particle-dominated phase of the universe which is in good agreement with [33],[60],[61].

 

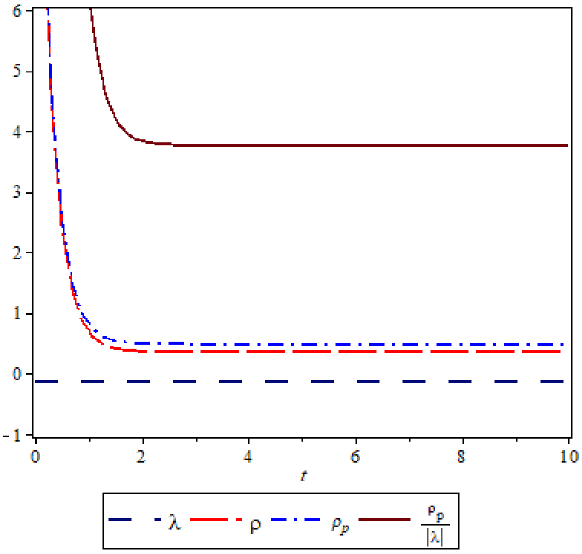


Figure 2: Variation of $\rho$, $\lambda$, $\rho_p$, and $\rho_p/|\lambda|$ Vs. $t$ for $\alpha = 0.4$, $\beta$=0.6, m=2.3, and $D_1 = 1.62$.

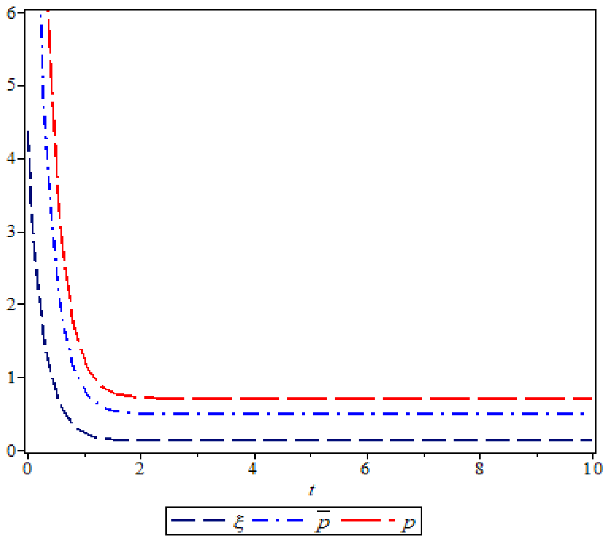


Figure 3: Variation of $p$, $\xi$, and $\bar{p}$ Vs. $t$ for $\alpha = 0.4$, $\beta$=0.6, m=2.3, and $D_1 = 1.62$.

From (26), and (27) we have obtained the coefficient of bulk viscosity as

$$\xi = \frac{\alpha^6\{4m^2 - 9\beta^2\} + D_1^2 e^{-6\beta t}}{48\pi\alpha^6}. \quad (53)$$

From (46), (53) into (22), the pressure can be obtained as

$$p = \frac{(1+\beta)\{4m^2\alpha^6 + D_1^2 e^{-6\beta t}\} + 9\alpha^6(1-\beta)\beta^2}{16\pi\alpha^6}. \quad (54)$$

From (13) the bulk viscous pressure can be obtained as

$$\bar{p} = \frac{\alpha^6\{4m^2 + 9\beta^2\} + D_1^2 e^{-6\beta t}}{16\pi\alpha^6}. \quad (55)$$

Fig.3 displays the behavior of the pressure $(p)$, the coefficient of bulk viscosity $(\xi)$, and the bulk viscous pressure $(\bar{p})$ versus cosmic time $t$. All the pressure $(p)$, the coefficient of bulk viscosity $(\xi)$, and the bulk viscous pressure $(\bar{p})$ have a large value in the beginning when $t = 0$ and decrease from positive with an increase in cosmic time $t$ to approach a constant value.

**CASE II: for $\gamma \neq 0$, $(q \neq -1)$**

We have obtained the metric coefficients $A, B,$ , and $C$ as

$$A = D_0,\ B = \frac{e^{\frac{\gamma D_1(Dt+\eta)}{2D(\gamma-3)(Dt+\eta)^{\frac{3}{\gamma}}}}}{D_3(Dt+\eta)^{-\frac{3}{2\gamma}}},\ \text{and } C = \frac{D_2(Dt+\eta)^{\frac{3}{2\gamma}}}{e^{\frac{\gamma D_1(Dt+\eta)}{2D(\gamma-3)(Dt+\eta)^{\frac{3}{\gamma}}}}}. \quad (56)$$

where $D_1$, $D_2$, and $D_3$ are constants.

Substituting $A, B,$ and $C$ from (56) in (1), we get

$$ds^2 = dt^2 - D_0^2 dx^2 - e^{2mx}\left\{ \frac{e^{\frac{\gamma D_1(Dt+\eta)}{D(\gamma-3)(Dt+\eta)^{\frac{3}{\gamma}}}}}{D_3^2(Dt+\eta)^{-\frac{3}{\gamma}}} dy^2 + \frac{D_2^2(Dt+\eta)^{\frac{3}{\gamma}}}{e^{\frac{\gamma D_1(Dt+\eta)}{D(\gamma-3)(Dt+\eta)^{\frac{3}{\gamma}}}}} dz^2 \right\}. \quad (57)$$

From (10) we have obtained the torsion scalar as

$$T = \frac{\gamma^2\left\{D_1^2(Dt+\eta)^{\frac{\gamma-3}{\gamma}} - 4m^2\right\}(Dt+\eta)^2 - 9D^2}{2\gamma^2(Dt+\eta)^2}. \quad (58)$$

Also, we have determined the volume $(V)$, the mean Hubble's parameter $(H)$, the expansion scalar $(\theta)$, the mean anisotropy parameter $(A_m)$, and the shear scalar $(\sigma^2)$ respectively as

$$V = (Dt+\eta)^{\frac{3}{\gamma}}, \quad (59)$$

$$H = \frac{D}{\gamma(Dt+\eta)}, \quad (60)$$

$$\theta = \frac{3D}{\gamma(Dt+\eta)}, \quad (61)$$

$$A_m = \frac{\gamma^2 D_1^2(Dt+\eta)^{\frac{2\gamma-6}{\gamma}} + 3D^2}{6D^2}, \quad (62)$$

$$\sigma^2 = \frac{\gamma^2 D_1^2(Dt+\eta)^{\frac{2\gamma-6}{\gamma}} + 3D^2}{4\gamma^2(Dt+\eta)^2}, \quad (63)$$

In Fig.4 the graphical representation of the volume $(V)$, the mean Hubble's parameter $(H)$, the expansion scalar $(\theta)$, the mean anisotropy parameter $(A_m)$, and the shear scalar $(\sigma^2)$ versus cosmic time $t$ has been depicted. One can see from Fig.4 that at an initial epoch when $t=0$ the volume $(V)$ of the universe is constant and starts increasing exponentially with an increase in cosmic time and diverges when $t\to\infty$, whereas the mean Hubble's parameter $(H)$, the expansion scalar, and the shear scalar $(\sigma^2)$ starts decreasing from $t=0$ and vanishes at a large time when $t\to\infty$, while the mean anisotropy parameter $(A_m)$ drastically decreases in the beginning and approaches to a constant value as $t\to\infty$. Also, the ratio $\sigma^2/\theta^2\neq 0$ shows that the constructed model doesn't approach isotropy.

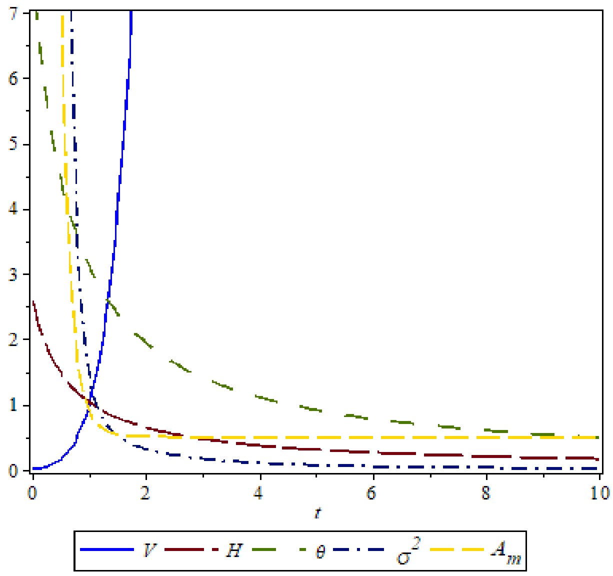

Figure 4: Variation of $V$, $H$, $\theta$, $A_m$, and $\sigma^2$ Vs. $t$ for $\eta=0.4$, $\beta$=0.8, m=2.3, $D=0.6$, and $D_1=1.62$.

From (24), we have obtained the value of energy density as

$$\rho=\frac{\gamma^2\left\{D_1^2(Dt+\eta)^{\frac{\gamma-3}{\gamma}}+4m^2\right\}(Dt+\eta)^2-9D^2}{16\pi\gamma^2(Dt+\eta)^2}. \tag{64}$$

From (21), and (22) we have obtained the value of tension density as

$$\lambda=\frac{3D^2(\gamma-3)}{8\pi\gamma^2(Dt+\eta)^2}. \tag{65}$$

Also, we have obtained the particle density as

$$\rho_p=\frac{\gamma^2\left\{D_1^2(Dt+\eta)^{\frac{\gamma-3}{\gamma}}+4m^2\right\}(Dt+\eta)^2-3D^2(2\gamma-3)}{16\pi\gamma^2(Dt+\eta)^2}. \tag{66}$$

In Fig.5 we have presented the physical behavior of energy density $(\rho)$, tension density $(\lambda)$, particle density $(\rho_p)$, and the comparative behavior $\rho_p/|\lambda|$ versus cosmic time $t$. At an initial epoch, energy density $(\rho)$ and particle density $(\rho_p)$ decreases rapidly from positive, and eventually both approach the same constant value, while the tension density $(\lambda)$ increases in negative and disappears over a large time. As $\lambda<0$ the string phase disappears in this model which is supported by [59], and the comparative behavior $\rho_p/|\lambda|<1$ implies that $\rho_p<|\lambda|$, attributing the string-dominated phase of the universe which is in good agreement with [33],[60],[61].

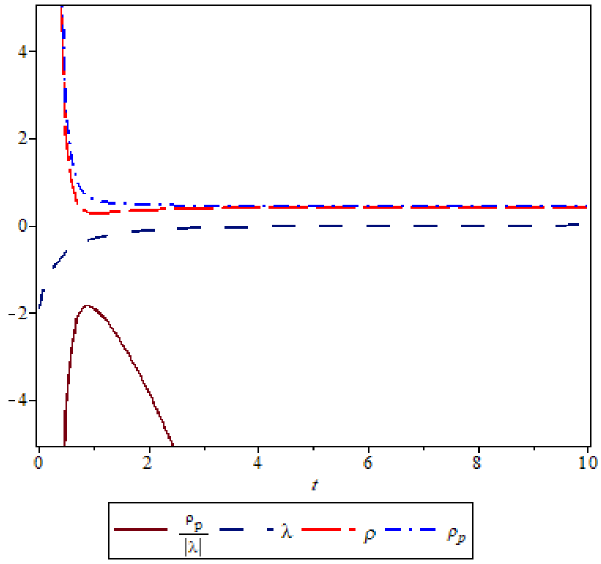

Figure 5: Variation of $\rho$, $\lambda$, $\rho_p$, and $\rho_p/|\lambda|$ Vs. $t$ for $\eta=0.4$, $\beta$=0.8, m=2.3, $D=0.6$, and $D_1=1.62$.

From (26), and (27) we have obtained the coefficient of bulk viscosity as

$$\xi=\frac{\beta\left\{\gamma^2\left(D_1^2(Dt+\eta)^{\frac{\gamma-3}{\gamma}}+4m^2\right)(Dt+\eta)^2-9D^2\right\}}{48\pi\gamma D(Dt+\eta)}. \tag{67}$$

From (60), (67) into (22), the pressure can be obtained as

$$p=\frac{\gamma^2(\beta+1)\left\{D_1^2(Dt+\eta)^{\frac{\gamma-3}{\gamma}}+4m^2\right\}(Dt+\eta)^2-3D^2\{2\gamma+3(\beta-1)\}}{16\pi\gamma^2(Dt+\eta)^2}. \tag{68}$$

From (13) the bulk viscous pressure can be obtained as

$$\bar{p}=\frac{\gamma^2\left\{D_1^2(Dt+\eta)^{\frac{\gamma-3}{\gamma}}+4m^2\right\}(Dt+\eta)^2-3D^2(2\gamma-3)}{16\pi\gamma^2(Dt+\eta)^2}. \tag{69}$$

 

The behavior of the pressure $(p)$, the coefficient of bulk viscosity $(\xi)$, and the bulk viscous pressure $(\bar{p})$ versus cosmic time $t$ has been depicted in Fig.6. Both the pressure $(p)$, and the bulk viscous pressure $(\bar{p})$ are decreases rapidly in the beginning and approach constant value at a large time $t \to \infty$, whereas the coefficient of bulk viscosity $(\xi)$ rapidly diminishes initially and float to infinity as $t \to \infty$.

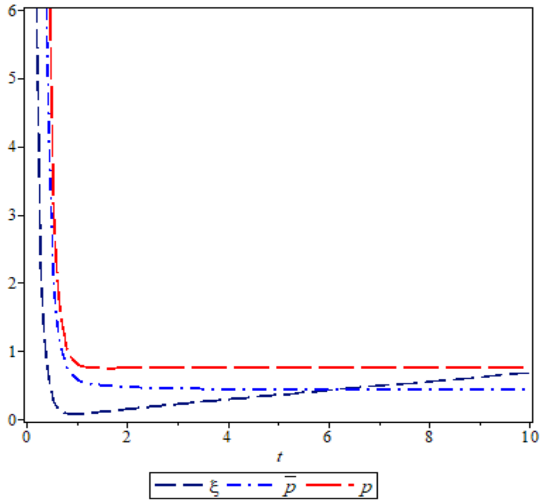


Figure 6: Variation of $p$, $\xi$, and $\bar{p}$ Vs. $t$ for $\eta = 0.4$, $\beta$=0.8, m=2.3, $D = 0.6$, and $D_1 = 1.62$.

## IV. Concluding Remarks

We have investigated the Bianchi-type V models which are spatially homogeneous and anisotropic in presence of bulk viscous fluid coupled with one-dimensional cosmic string in teleparallel gravity. We have discussed two different models, one obtained from the average scale factor $a = \alpha e^{\beta t}$ showing the solutions for the deceleration parameter $q = -1$, for $\gamma = 0$, in which the model obtained in (43) represents the particle-dominated universe and another for $a = (Dt + \eta)^{\frac{1}{\gamma}}$ showing the solutions for the deceleration parameter $q = \gamma - 1$, for $\gamma \neq 0$ and the model found in (57) is the string-dominated universe, both are in good agreement with [33],[60],[61]. The derived models are anisotropic, and the string phase disappears as tension density is negative which is supported by [59]. In both cases, we have discussed the kinematical and geometrical properties and interestingly our constructed models resemble with the recent observational data.

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