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BULK VISCOUS STRING COSMOLOGICAL MODEL WITH POWER LAW VOLUMETRIC EXPANSION IN TELEPARALLEL GRAVITY

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In this paper, we have investigated the Bianchi-type V cosmological model which is spatially homogeneous and anisotropic in presence of bulk viscous fluid containing one-dimensional cosmic string. We have obtained the exact solutions of highly non-linear differential field equations considering the power-law volumetric expansion of the universe and $f(T) = T$ formalism. Some physical and kinematical properties of the constructed model have been discussed and presented graphically and it is interesting to note that the resultant model resembles the recent observational data.

Keywords: *bulk viscous fluid: cosmic string: teleparallel gravity*

1. Introduction

Recent observations and measurements from high redshift supernovae [1-3] indicate that the universe is accelerating. The cause of the universe's acceleration is unknown; it is commonly referred to as the dark energy problem, which is caused by the universe's negative pressure. Two approaches have been proposed to address this issue: one is to develop viable dark energy models, while the other modify Einstein's gravitation theory. Nojiri & Odintsov [4] has reviewed various modified gravities and considered a gravitational alternative for dark energy. Again Nojiri et al. [5,6] reviewed some standard issues and discussed some latest developments in modified gravity as well

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as unified cosmic history in modified gravity.

Numerous modified gravity theories exist to investigate the unknown and hidden aspects of the universe. Amongst them, the $f(T)$ theory of gravitation, which is based on a modification of the teleparallel equivalent of general relativity, is a viable candidate. Many researchers have discussed various aspects of $f(T)$ gravity. Cai et. al. [7] provided a brief review of $f(T)$ gravity and cosmology. Myrzakulov [8] has studied the accelerating universe from $f(T)$ gravity. Numerous cosmologists have developed theoretical cosmological models that behave similarly to the present physical universe, which is anisotropic, expanding, and accelerating. Pawar & Dabre [9] have studied an anisotropic string cosmological model for perfect fluid distribution in $f(T)$ gravity. Chirde & Shekh [10] examined the thermodynamical aspect of barotropic bulk viscous fluid in teleparallel gravity. Sharif & Rani [11] studied bulk viscosity taking dust matter in generalized teleparallel gravity. Sadatian [12] analyzed the effect of viscous content on the modified cosmological $f(T)$ model.

Various cosmologists and physicists have studied the theoretical development of the universe and the effects of bulk viscosity and string on cosmic evolution using the source as a bulk viscous fluid containing a string of clouds. Mishra & Dua [13], investigated the dynamics of the universe for bulk viscous string cosmological model using the LRS Bianchi type II metric in the Saez-Ballester theory of gravitation. Tripathy et al., [14] studied LRS Bianchi I model in reference to Einstein's relativity using the source as stiff viscous fluid coupled with an electromagnetic field. Kiran & Reddy [15] presented the non-existence of Bianchi type III bulk viscous string cosmological model in $f(R,T)$ gravity. Santhi et al, [16,17] investigated bulk viscous string cosmological models using Bianchi type II, VIII, IX, and VI_h space-times in $f(R)$ gravity. Pawar & Dabre [18] studied the bulk viscous string cosmological model using the special law of variation for Hubble's parameter in teleparallel gravity. Using the Kantowski-Sachs metric, Reddy et al., [19] built an isotropic bulk viscous string cosmological model that illustrates the special case for non-validating cosmic strings. Hegazy, [20] devised a formula for calculating cosmic entropy in terms of viscosity and applied it to investigate the entropy, enthalpy, Gibbs energy, and Helmholtz energy of a constructed model in the presence of viscosity. Naidu et al., [21-26] vigorously investigated bulk viscous string cosmological models in relation to different gravitational theories. Several cosmologists [27-33] have obtained some recent and significant investigations of bulk viscous fluid in the presence of cloud strings in various contexts.

Motivated by the situations discussed above in this paper, we have considered spatially homogeneous and anisotropic Bianchi type V space-time to construct the bulk viscous string cosmological model within the context of teleparallel gravity. This paper is divided into several sections: Sec. 2 deals with elementary definitions and equations of motion in the framework of teleparallel gravity. In Sec. 3 considering spatially homogeneous and anisotropic Bianchi type V metric, we have obtained the corresponding field equations. In Sec. 4, we have obtained the exact solution of highly non-linear field equations along with different physical and kinematical quantities and presented them with 3D graphs. Lastly, in Sec. 5, we have concluded the investigations.

2. Elementary definitions and equation of motion

In this section, we provide a concise explanation of $f(T)$ gravity and a thorough derivation of its field equations. The line element for a general space-time is defined as

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} , \tag{1}$$

where $g_{\mu\nu}$ are the components of the metric tensor which are symmetric. The above line element can be transformed into the Minkowskian space-time (which represents the dynamic fields of the theory) as

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = \eta_{ij} \theta^i \theta^j , \tag{2}$$

$$dx^{\mu} = e_i^{\mu} \theta^i , \quad \theta^i = e_{\mu}^i dx^{\mu} , \tag{3}$$

where η_{ij} is a metric tensors in Minkowskian space-time such that $\eta_{ij} = \mathrm{diag}[1, -1, -1, -1]$ and $e_i^{\mu} e_{\nu}^i = \delta_{\nu}^{\mu}$ or $e_i^{\mu} e_{\mu}^j = \delta_i^j$. $\sqrt{-g} = \det[e_{\mu}^i] = e$ and the dynamic fields of the theory are represented by the tetrads matrix e_{μ}^{α}. The Weitzenbocks connection components which have a zero curvature but nonzero torsion for a manifold are defined as

$${}^{\alpha}_{\mu\nu} = e_i^{\alpha} \partial_{\nu} e_{\mu}^i = - e_{\mu}^i \partial_{\nu} e_i^{\alpha} . \tag{4}$$

The components of the torsion tensor for a manifold are defined by the anti-symmetric part of the Weitzenbocks connection

$$T^{\alpha}_{\mu\nu} = {}^{\alpha}_{\mu\nu} - {}^{\alpha}_{\nu\mu} = e_i^{\alpha} \left(\partial_{\mu} e_{\nu}^i - \partial_{\nu} e_{\mu}^i \right). \tag{5}$$

Con-torsion tensor components are defined by

$$K_{\alpha}^{\mu\nu} = -\frac{1}{2} \left(T_{\alpha}^{\mu\nu} - T_{\alpha}^{\nu\mu} - T_{\alpha}^{\mu\nu} \right). \tag{6}$$

A new tensor, $S_{\alpha}^{\mu\nu}$ constructed from the components of the torsion and con-torsion tensors for a better understanding of the definition of the scalar equivalent to the curvature scalar of Riemannian geometry as follows,

$$S_{\alpha}^{\mu\nu} = \frac{1}{2} \left(K_{\alpha}^{\mu\nu} + \delta_{\alpha}^{\mu} T_{\beta}^{\beta\nu} - \delta_{\alpha}^{\nu} T_{\beta}^{\beta\mu} \right). \tag{7}$$

The torsion scalar is defined using the contraction which is similar to the scalar curvature in general relativity as

$$T = T^{\alpha}_{\mu\nu} S^{\mu\nu}_{\alpha} . \tag{8}$$

The action is defined by generalizing the teleparallel gravity, i.e, $f(T)$ theory as

$$S = \int [f(T) + L_{Matter}] e d^4 x , \tag{9}$$

where $f(T)$ denotes an algebraic function of the torsion scalar *T*.

Equations of motion are obtained by functional variation of the action (9) with respect to the tetrads as

$$S^{\nu\rho}_{\mu} \partial_{\rho} T f_{TT} + \left[e^{-1} e^{i}_{\mu} \partial_{\rho} \left(e e^{\alpha}_{i} S^{\nu\rho}_{\alpha} \right) + T^{\alpha}_{\lambda\mu} S^{\nu\lambda}_{\alpha} \right] f_T + \frac{1}{4} \delta^{\nu}_{\mu} f = 4\pi T^{\nu}_{\mu} , \tag{10}$$

where the energy-momentum tensor T^{ν}_{μ} is considered as bulk viscous fluid with one-dimensional cosmic string, f_T and f_{TT} denotes respectively the first and second-order derivatives of $f(T)$ with respect to *T*. For $f(T) = \text{const}$, the equations of motion in (10) reduce to the equations of motion of the teleparallel gravity with a cosmological constant, which is dynamically equivalent to general relativity. These equations depend on the choice made for the set of tetrads.

3. Metric and field equations

We consider a line element

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx} \left(B^2 dy^2 + C^2 dz^2 \right), \tag{11}$$

which is a spatially homogeneous and anisotropic Bianchi type-V metric in which *m* is constant and *A*, *B*, and *C* are a function of cosmic time *t* only.

Consider the set of diagonal tetrads related to the metric (11) as

$$\left[e^{\nu}_{\mu} \right] = \text{diag} \left[1, A, B e^{mx}, C e^{mx} \right]. \tag{12}$$

Then the determinant of the matrix (11) is

$$e = ABC e^{2mx} . \tag{13}$$

The torsion scalar (8) is obtained as

$$T=-2\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+m^2\right). \tag{14}$$

We consider the source as bulk viscous fluid containing one-dimensional cosmic string given by

$$T_\mu^\nu=(\rho+\bar{p})u_\mu u^\nu+\bar{p}g_\mu^\nu-\lambda x_\mu x^\nu , \tag{15}$$

$$\bar{p}=p-3\xi H , \tag{16}$$

where $\rho=\rho_p+\lambda$ is the proper string energy density with particles attached to them and ρ_p is the particle energy density, λ is the strings tension density, $3\xi H$ is bulk viscous pressure, $\xi(t)$ is the coefficient of bulk viscosity, H is Hubble's parameter, x^ν denotes a unit space-like vector for the cloud string and u^ν denotes four-velocity vector satisfying the conditions, $u^\nu u_\nu=-1=-x^\nu x_\nu$ and $u_\nu x^\nu=0$.

In a co-moving coordinate system, we have

$$u^\nu=(0,0,0,1), \quad x^\nu=\left(A^{-1},0,0,0\right). \tag{17}$$

We obtained the field equations for Bianchi type-V space-time (11), from (10) and (15)-(16) in the framework of teleparallel gravity as

$$f+2f_T\left(\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+2m^2\right)+2\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}=16\pi(p-3\xi H-\lambda), \tag{18}$$

$$f+2f_T\left(\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+2\frac{\dot{A}\dot{C}}{AC}+2m^2\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}=16\pi(p-3\xi H), \tag{19}$$

$$f+2f_T\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+2m^2\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=16\pi(p-3\xi H), \tag{20}$$

$$f+4f_T\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right)=-16\pi\rho , \tag{21}$$

$$\left(2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)f_T=0 , \tag{22}$$

$$\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)f_T=0, \tag{23}$$

where the overhead dot (.) denotes the derivative with respect to cosmic time t.

By solving (22) and (23) above field equations reduces to

$$f+2f_T\left(\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+2\frac{\dot{B}\dot{C}}{BC}+2m^2\right)+2\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}=16\pi(p-3\xi H-\lambda), \tag{24}$$

$$f+2f_T\left(\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+2m^2\right)+2\left(\frac{\dot{C}}{C}\right)\dot{T}f_{TT}=16\pi(p-3\xi H), \tag{25}$$

$$f+2f_T\left(\frac{\ddot{B}}{B}+\frac{\dot{B}\dot{C}}{BC}+2m^2\right)+2\left(\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=16\pi(p-3\xi H), \tag{26}$$

$$f+4f_T\left(\frac{\dot{B}\dot{C}}{BC}\right)=-16\pi\rho. \tag{27}$$

Thus, we have four non-linear differential equations with seven unknowns, namely f, B, C, p, ξ, ρ, and λ; solutions which are discussed in the next section.

4. Solutions of field equations

As there are four highly non-linear differential equations (24)-(27) and seven unknowns, in order to obtain the exact solutions we consider the linear $f(T)=T$ gravity along with the special power-law volumetric expansion of the universe as

$$V=t^{3n}. \tag{28}$$

where n is a non-zero constant.

We find some kinematical space-time quantities of physical interest in cosmology.

The spatial volume V is defined as

$$V=D_0BC. \tag{29}$$

where D_0 is an integrating constant.

Also, the volumetric expansion rate of the universe is described by the generalized mean Hubble's parameter H given by

$$H = \frac{1}{3}\sum_{i=1}^{3} H_i = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{30}$$

in which H_1, H_2, H_3 denotes the directional Hubble parameters.

From Eqs. (29) and (30), we get

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\sum_{i=1}^{3} H_i \,. \tag{31}$$

To analyze, whether the model approaches isotropy or not, we discuss the mean anisotropy parameter A_m, as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right)^2 . \tag{32}$$

The expansion scalar θ and the shear scalar σ^2 are respectively defined as

$$\theta = u^{\mu}_{;\mu} = 3H \,, \tag{33}$$

$$\sigma^2 = \frac{3}{2} A_m H^2 \,, \tag{34}$$

The deceleration parameter is defined as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{35}$$

We obtained the metric coefficients A, B, and C as

$$A = D_0 \,, \quad B = \frac{\sqrt{t^{3n}}}{D_3 e^{D_1 t / 2(3n-1) t^{3n}}} \,, \quad C = D_2 \sqrt{t^{3n}} \; e^{D_1 t / 2(3n-1) t^{3n}} \,. \tag{36}$$

where D_1, D_2 and D_3 are constants.

Substituting A, B, and C from (36) in (11), we get

$$ds^2 = dt^2 - D_0^2 dx^2 - e^{2mx}\left(\frac{t^{3n}}{D_3^2 e^{D_1 t/(3n-1)t^{3n}}} dy^2 + D_2^2 t^{3n} e^{D_1 t/(3n-1)t^{3n}} dz^2\right). \tag{37}$$

From (14) we have obtained the torsion scalar as

$$T = -\frac{4m^2t^2 - D_1^2 t^{2-6n} + 9n^2}{2t^2}. \tag{38}$$

Also, we have determined the mean Hubble's parameter H, the expansion scalar θ, the mean anisotropy parameter A_m, the shear scalar σ^2, and the deceleration parameters q respectively as

$$H = \frac{n}{t}, \tag{39}$$

$$\theta = \frac{3n}{t}, \tag{40}$$

$$A_m = \frac{D_1^2 t^{2-6n} + 3n^2}{6n^2}, \tag{41}$$

$$\sigma^2 = \frac{D_1^2 t^{2-6n} + 3n^2}{4t^2}, \tag{42}$$

$$q = \frac{1-n}{n} = \text{const}. \tag{43}$$

The graphical representation of Hubble's parameter H versus cosmic time t is depicted in Fig.1, where at an initial epoch when $t = 0$ with an increasing value of n the value of H increases and get vanishes as $t \to \infty$. This shows that the expansion of the universe is getting faster with an increasing value of n but becomes slower with increasing cosmic time t. The ratio $\sigma^2/\theta^2 \neq 0$ shows the constructed model doesn't approach isotropy. Also, the sign of q in (43) demonstrates whether the model is accelerating or not. The positive sign of q i.e. for $0 \le n \le 1$ corresponds to a plain decelerating cosmological model although the deceleration parameter in range $-1 \le q < 0$ corresponds to an accelerating universe and for $q = 0$ i.e. for $n = 1$ corresponds to the evolution with a constant rate. The observational

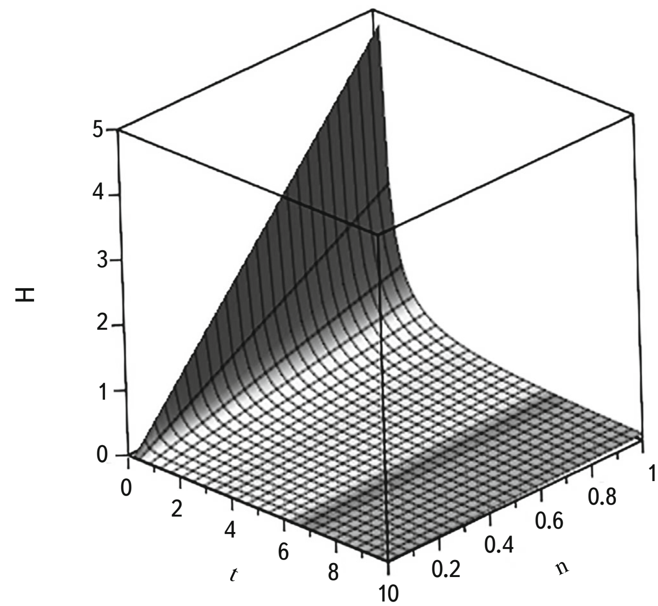

Fig.1. Variation of H vs. t.

evidences [1,2] supports the accelerating phase of the universe i.e $-1 \le q < 0$.

From (27) we obtained the value of energy density as

$$\rho = \frac{4m^2t^2 + D_1^2 t^{2-6n} - 9n^2}{32\pi t^2}. \tag{44}$$

Solving (24) and (25), we have obtained the value of tension density as

$$\lambda = -\frac{3n(3n-1)}{16\pi t^2}. \tag{45}$$

Also, we have obtained the particle density as

$$\rho_p = \frac{4m^2t^2 + D_1^2 t^{2-6n} + 3n(3n-2)}{32\pi t^2}. \tag{46}$$

Fig.2 depicts the variation of energy density ρ versus cosmic time t, in which the energy density is very small in the starting phase of evolution for both varying constant n and cosmic time t but as both increases, the energy density becomes a decreasing function of cosmic time t. Whereas the representation of tension density λ as shown in Fig.3 shows that initially, tension density diminishes from positive but with an increasing n it shows the transition from positive to negative for tension density to grow in negative and get vanish when $t \to \infty$. For a small period

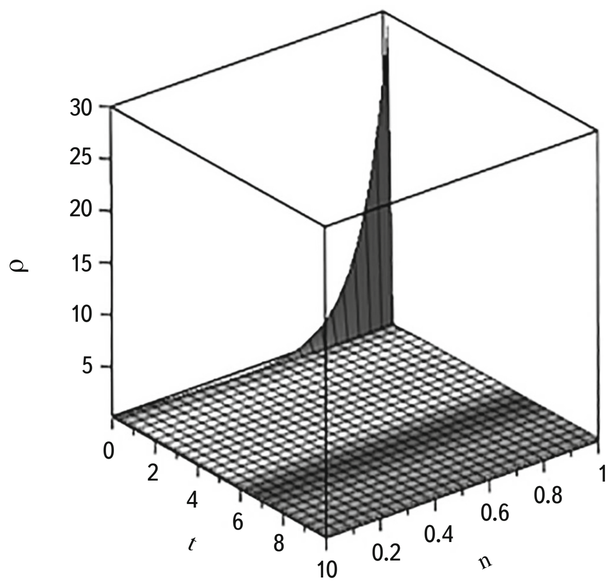

Fig.2. Variation of ρ vs. t.

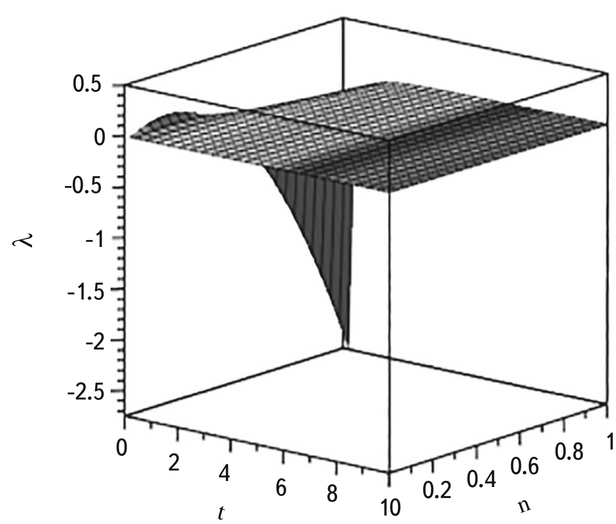

Fig.3. Variation of λ vs. t.

of n, the tension density is positive i.e. $\lambda \geq 0$ showing the presence of strings in the universe while after the transition the tension density $\lambda < 0$ showing the string phase disappears which is supported by [34].

We assume that the coefficient of viscosity should vary with the expansion scalar in such a way that

$$\xi\theta = \xi_0 = \text{const}. \tag{47}$$

From (47) we have obtained the coefficient of bulk viscosity as

$$\xi = \frac{\xi_0 t}{3n}. \tag{48}$$

From (26) the pressure can be obtained as

$$p = \frac{4\left(m^2 + 8\pi\xi_0\right)t^2 + t^{2-6n} D_1^2 + 3n\left(3n-2\right)}{32\pi t^2}. \tag{49}$$

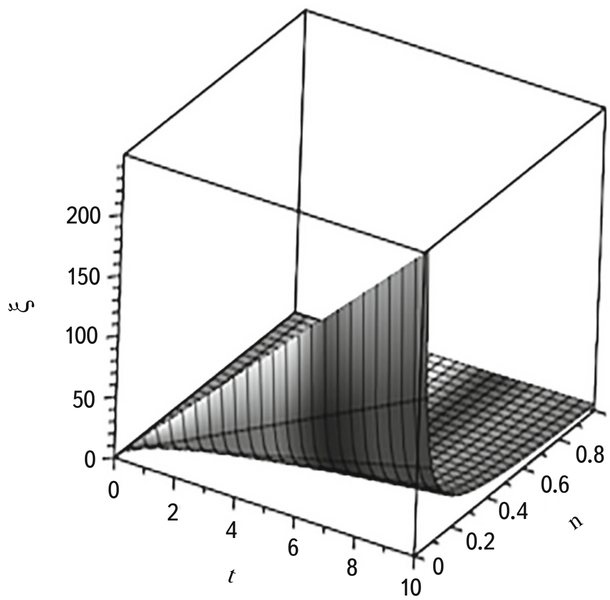

Fig.4. Variation of ξ vs. t.

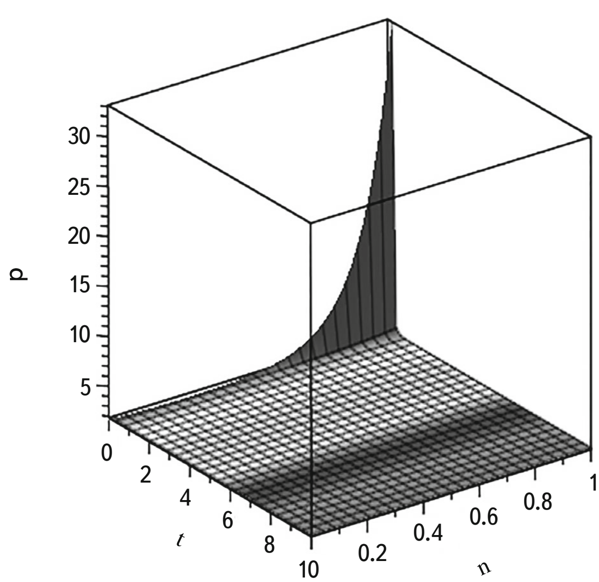

Fig.5. Variation of p vs. t.

It is seen from Fig.4 that the coefficient of bulk viscosity is an increasing function of cosmic time t for small n but with an increasing n the value of ξ becomes steady. While the pressure is incredibly small for a small value of n but as n increases the pressure diminishes from positive to approach constant with an increasing cosmic time t (Fig.5).

5. Concluding remarks

In this paper, we have studied the spatially homogeneous and anisotropic Bianchi type V bulk viscous string cosmological model within the context of teleparallel gravity. The deceleration parameter is obtained to be a constant value that shows the decelerating or accelerating phase of the universe depending on the value of n. The Hubble's parameter shows the expansion of the universe is getting faster in the beginning with varying n and become slower through time. Also, the constructed model is purely anisotropic. Energy density is positive throughout the expansion whereas we have found the presence of string in an initial phase but later on string phase disappears which is supported by [34]. The coefficient of bulk viscosity shows transference with varying n and pressure becomes a diminishing function of cosmic time t with increasing n.

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