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Research Paper

Accelerating Expansion of the Universe with Dark Matter and Holographic Dark Energy in *f(T)* Gravity

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Abstract— This paper deals with the study of spatially homogeneous and anisotropic Bianchi-type V space-time filled with dark matter and holographic dark energy in the framework of *f(T)* gravity by considering *f(T)=T* formalism. In order to solve the gravitational field equations completely, the exponential law of volumetric expansion has been employed. The derived cosmological model is flat and free from any kind of singularities. Some cosmologically important physical and kinematical parameters of the model have been obtained, and discussed with the help of their graphical representations. It is observed that the universe has accelerating expansion; it is stable throughout the expansion, and physically acceptable. Notably, the resultant model is consistent with the recent observations.

Keywords— Bianchi-type V space-time, Anisotropy, Dark Matter, Holographic Dark Energy, Stability, f(T) Gravity.

1. Introduction

The recent cosmological observations specify that the present universe is undergoing an accelerating expansion. The measurements from high redshift supernovae experiments provide direct evidence for an accelerating cosmic expansion [1-7]. However, the cause of this accelerating cosmic expansion is yet to achieve satisfactorily. Several theories have been proposed by the researchers in order to investigate the cause of cosmic acceleration. Astrophysical observations in view of cosmic expansion points towards the presence of some kind of repulsive force in the universe that is repelling the cosmic objects farther apart, and suggest that this cosmic acceleration is driven by mysterious kind of energy with a large negative pressure, termed as dark energy (DE). In the literature, various types of DE candidates are proposed, and in this regard, many researchers have constructed various DE models such as quintessence [8-10], quintom [11, 12], phantom [13], *k*-essence [14, 15], and tachyon [16] as well as holographic dark energy (HDE) [17-20], Ricci DE [21], new age graphic DE [22, 23], Chaplygin gas [24], extended Chaplygin gas [25, 26] and the generalized Chaplygin gas [27, 28], etc., to fulfill their curiosity in studying expansion cosmological expansion.

Cosmological measurements based on the Planck data [29] on the cosmic microwave background radiation (CMBR) indicate that 70% our universe is filled with DE and 30% of it with non-relativistic matter, viz., baryons and dark matter (DM). There are few particles termed DM candidates [30-32], viz., weakly interacting massive particles (WIMPs), SM neutrinos, sterile neutrinos, axions, and super symmetric (SUSY) candidates (neutralinos, neutrinos, gravitinos, axinos). The WIMPs consist of the lightest SUSY particles, particularly, the neutralinos, which is considered as the most likely candidate of DM. Axions are found to be additionally high candidates of DM. The non-inclusion of DM particle yet in the table of elementary cosmic particles means its true nature is still unknown. Usually, the DE component is characterized by dynamically variable equation of state (EoS) parameter which is equal to the ratio of spatially homogeneous pressure to the energy density of DE. To explain the accelerating expansion of the universe, two methods have been proposed; one is to investigate various dark energy candidates and the other is to modify the general theory of relativity (GTR).

Since the GTR is not enough to justify the gravitational interactions, other theories, such as alternating theories and more precisely modified theories of gravitation such as *f(R)*, *f(T)*, *f(R, T)*, *f(G)*, *f(R, G)*, *f(Q)*, *f(T, B)*, etc., are developed to investigate the unknown and hidden aspects of the universe. Among these numerous modified gravity theories, *f(T)* theory of gravitation is a feasible candidate. The *f(T)* theory of gravitation is based on a modification of the teleparallel equivalent of GTR. Several researchers have investigated various aspects of different cosmological models such as FRW, Bianchi types, Kaluza-Klein, Kantowski-Sach, Einstein-Rosen, etc., with different types of matter source in the framework of *f(T)* gravity. Among various approaches for describing accelerating cosmic expansion or dark sector of the universe, the holographic dark energy (HDE) models have received immense attention of the researchers. The notion of HDE arose from an attempt of applying the holographic principle (HP) to the DE problem, which states that all of the information contained in a volume of space can be represented as a hologram. HDE model is a quantum gravity approach to the DE problem.

The anisotropy and homogeneity play a convincing role in the early stage of cosmic evolution since the course of action of isotropization of the universe becomes viable in the passage of time, and hence the study of spatially homogeneous and anisotropic models of the universe gain importance. The spatial homogeneity, anisotropy and simplicity of field equations, the Bianchi type cosmological models gained importance. Hence, many authors assume Bianchi type cosmological models to obtain exact solutions of the field equations of GTR and various alternative theories of gravitation.

Motivated by the above discussion and the work carried out by the researchers, we have taken up the study of spatially homogeneous and anisotropic Bianchi type V cosmological model in presence of DM and HDE in the framework of *f(T)* gravity. We have considered *f(T)=T* formalism and in order to solve the field equations completely, we employed the exponential law of volumetric expansion. The physical behavior of the constructed model is studied through the graphical representations of some parameters of cosmological importance, such as pressure, energy density, EoS parameter, volume, Hubble's parameter, expansion scalar, shear scalar, anisotropy parameter, squared speed of sound, etc. The value of deceleration parameter shows the accelerating expansion of the universe. Also, the obtained model is flat and singularity free; stable throughout the expansion and physically acceptable, and hence consistent with the recent observations.

This paper is divided as follows: Section 2 contains the related work for the study carried out in this paper, Section 3 provides the methodology of *f(T)* gravity in brief, Section 4 deals with the metric and derivation of its field equations, Section 5 contain exact solution of the field equations and construction of the cosmological model, Section 6 contain the derivation of some physical and kinematical parameters along with stability factor and their graphical representations with discussion and Section 7 concludes the research work.

2. Related Work

Initially, *f(T)* gravity theory was proposed to study cosmic inflation [33, 34], then accelerating cosmic expansion [35-38], its perturbations [39-41], and its reconstructions [42- 46]. Later on Ferraro [47], succinctly reviewed the *f(R)* and *f(T)* gravity theories and described some remarkable applications to cosmology and cosmic strings. Böhmer et.al. [48], studied the entity of relativistic stars and specifically constructed classes of solutions for a static perfect fluid. Pawar et al. [49],[50] studied string-coupled cosmological models using the Bianchi type V and VI_0 space-times in modified gravity. Zhang et al., [51] investigated the evolution of *f(T)* theory of gravity by considering logarithmic and power law form of *f(T)* models. Li et al., [52] studied generalized teleparallel gravity with local Lorentz invariance assuming the case *f(T)=T*. Sharif & Rani [53] studied the evolution of universe using ideal perfect fluid by considering three different forms *f(T)* gravity model and discussed the viscosity effect on the accelerating cosmic expansion. Similarly, due to the curiosity for scientific exploration of the universe, many cosmologists have taken remarkable efforts in *f(T)* theory of gravity by constructing different cosmological models [54-61].

In recent years, the interesting observations made in determining the nature of dark energy in quantum gravity termed as holographic dark energy (HDE) [62, 63] is receiving immense attention for researchers. Granda and Oliveros [64] with the use of infra-red cut-off, showed that the HDE model represents an accelerated cosmic expansion and its consistency with current observational data. Setare et al. [65, 66] have studied cosmological application of HDE density in non-flat universe. Adhao et al. [67], studied the anisotropic Bianchi type-I cosmological model with interacting DM and HDE in two cases for the values of deceleration parameter. They have shown that there is no coincidence problem, the models attain isotropy after some finite time, and the universe is undergoing accelerating expansion. Chirde and Shekh [68] studied the minimally interacting LRS Bianchi type-I power law and exponential law HDE models in f(T) gravity, and shown the accelerated cosmic expansion in both the cases. Mete et al. [69] studied the minimally interacting DM and HDE model in scalar tensor theory formulated by Saez and Ballester. Dubey et al. [70] constructed the Renyi HDE interacting flat FLRW model in the framework of Brans-Dicke theory of gravitation by considering infra-red cut-off as the Hubble's horizon, and shown that the model is stable at the late time. Shekh and Ghaderi [71] constructed and analyzed an interacting DM and HDE model, and they showed an accelerating expansion of the universe throughout the evolution.

3. Methodology *f(T)* Gravity: a Brief Review

Here, we make a discussion on the methodology of *f(T)* gravity and derivation of its field equations in brief. The general space-time metric is defined as

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta, \tag{1}$$

where $g_{\alpha\beta}$ are the components of the fundamental metric tensor. The line element (1) can be transformed through the Minkowski's description of the transformation, in the form:

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \eta_{hk}\theta^h\theta^k, \tag{2}$$

$$dx^{\alpha} = e_h^{\alpha}\theta^h, \quad \theta^h = e_{\alpha}^h dx^{\alpha}, \tag{3}$$

where η_{hk} are the components of metric tensors in Minkowskian space-time such that $\eta_{hk} = diag[1,-1,-1,-1]$ and $e_h^{\alpha}e_{\beta}^h = \delta_{\beta}^{\alpha}$ or $e_h^{\alpha}e_{\alpha}^k = \delta_h^k$. $\sqrt{-g} = \det[e_{\alpha}^h] = e$ and the tetrad matrix e_{μ}^{α} represents the dynamic fields of the theory. The components of Weitzenbocks connection which have a zero curvature and nonzero torsion for a manifold are defined as

$$\Gamma^{\gamma}{}_{\alpha\beta} = e_h^{\gamma}\partial_{\beta}e_{\alpha}^h = -e_{\alpha}^h\partial_{\beta}e_h^{\gamma}. \tag{4}$$

The torsion tensor for a manifold is defined by the anti-symmetric part of the Weitzenbocks connection as,

$$T^{\gamma}{}_{\alpha\beta} = \Gamma^{\gamma}{}_{\alpha\beta} - \Gamma^{\gamma}{}_{\beta\alpha} = e_h^{\gamma}(\partial_{\alpha}e_{\beta}^h - \partial_{\beta}e_{\alpha}^h). \tag{5}$$

The Con-torsion tensor which is the difference between the Levi-Civita and Weitzenbocks connection, is defined by

$$K^{\alpha\beta}{}_{\gamma} = -\frac{1}{2}\left(T^{\alpha\beta}{}_{\gamma} - T^{\beta\alpha}{}_{\gamma} - T_{\gamma}{}^{\alpha\beta}\right). \tag{6}$$

A new tensor, $S_{\alpha}{}^{\mu\nu}$, is constructed from the torsion and con-torsion tensors for a better understanding of the definition of the scalar equivalent to the curvature scalar of Riemannian geometry as follows,

$$S_{\gamma}{}^{\alpha\beta} = \frac{1}{2}\left(K^{\alpha\beta}{}_{\gamma} + \delta_{\gamma}^{\alpha}T^{\mu\beta}{}_{\mu} - \delta_{\gamma}^{\beta}T^{\mu\alpha}{}_{\mu}\right). \tag{7}$$

The torsion scalar which is similar to the scalar curvature in general relativity is defined by using the contraction as

$$T = T^{\gamma}{}_{\alpha\beta}S_{\gamma}{}^{\alpha\beta}. \tag{8}$$

The action is defined by generalizing the teleparallel gravity, i.e, *f(T)* theory as

$$S = \int [f(T) + L_{matter}]ed^4x, \tag{9}$$

where $f(T)$ is an algebraic function of the torsion scalar T .

On making the functional variation of the action (9) with respect to the tetrads, the equations of motion are obtained as

$$S_{\alpha}{}^{\beta\rho}\partial_{\rho}Tf_{TT} + \left[e^{-1}e_{\alpha}^h\partial_{\rho}\left(ee_h^{\gamma}S_{\gamma}{}^{\beta\rho}\right) + T^{\gamma}{}_{\lambda\alpha}S_{\gamma}{}^{\beta\lambda}\right]f_T + \frac{1}{4}\delta_{\alpha}^{\beta}f = 4\pi\left\{T_{\alpha}^{\beta} + \bar{T}_{\alpha}^{\beta}\right\}, \tag{10}$$

where T_{α}^{β} and $\bar{T}_{\alpha}^{\beta}$ are the energy-momentum tensors for DM and HDE respectively, $f_T = \frac{df(T)}{dT}$, $f_{TT} = \frac{d^2f(T)}{dT^2}$, and by setting $f(T) = a_0 =$ constant, the equations (10) are the same as that of the teleparallel gravity equivalent to the General Relativity.

4. Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-V space-time in the form

$$ds^2 = dt^2 - X^2dx^2 - e^{2mx}\left(Y^2dy^2 + Z^2dz^2\right), \tag{11}$$

where *m* is a constant and *X, Y,* and *Z* are functions of cosmic time *t* only.

Then the set of diagonal tetrads related to the metric (11) is

$$[e_{\alpha}^{\beta}] = diag[1, X, Ye^{mx}, Ze^{mx}] \tag{12}$$

and the determinant of a matrix (12) is

$$e = XYZe^{2mx}. \tag{13}$$

The torsion scalar (8) for the space-time (11) is obtained as

$$T = -2\left(\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{X}\dot{Z}}{XZ} + m^2\right). \tag{14}$$

The energy-momentum tensor for DM and HDE are respectively given by [68]

$$T_{\alpha}^{\beta} = \rho_m u_{\alpha}u^{\beta}, \tag{15}$$

and

$$\bar{T}_{\alpha}^{\beta} = (\rho_{\Lambda} + p_{\Lambda})u_{\alpha}u^{\beta} - p_{\Lambda}\delta_{\alpha}^{\beta}, \tag{16}$$

together with co-moving coordinates satisfying

$$u^{\beta} = (0,0,0,1) \text{ and } u_{\alpha}u^{\alpha} = 1. \tag{17}$$

where u^{β} is the four-velocity vector of the fluid, ρ and p are respectively the energy density and pressure of the fluids.

We obtained the field equations for (11), using (15) and (16) in (10) as

$$f + 2f_T\left(\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Y}}{XY} + 2\frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{X}\dot{Z}}{XZ} + 2m^2\right) + 2\left(\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right)\dot{T}f_{TT} = -16\pi p_{\Lambda}, \tag{18}$$

$$f + 2f_T\left(\frac{\ddot{X}}{X} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + 2\frac{\dot{X}\dot{Z}}{XZ} + 2m^2\right) + 2\left(\frac{\dot{X}}{X} + \frac{\dot{Z}}{Z}\right)\dot{T}f_{TT} = -16\pi p_{\Lambda}, \tag{19}$$

$$f + 2f_T\left(\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + 2\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{X}\dot{Z}}{XZ} + 2m^2\right) + 2\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y}\right)\dot{T}f_{TT} = -16\pi p_{\Lambda}, \tag{20}$$

$$f + 4f_T\left(\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{X}\dot{Z}}{XZ}\right) = 16\pi\left(\rho_m + \rho_{\Lambda}\right), \tag{21}$$

$$\left(2\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} - \frac{\dot{Z}}{Z}\right)f_T = 0, \tag{22}$$

$$\left(\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right)f_T = 0, \tag{23}$$

where the overhead dot (.) denotes the derivative with respect to cosmic time *t*.

Equations (22) and (23) give

$$A = constant,\ k\ (say). \quad (24)$$

We consider *f(T)* formalism as *f(T)*= *T* . Then, using (14) and (24) in (18) –(21), we obtain

$$-8\pi p_\Lambda = \frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}\dot{Z}}{YZ} + m^2 \quad (25)$$

$$-8\pi p_\Lambda = \frac{\ddot{Z}}{Z} + m^2 \quad (26)$$

$$-8\pi p_\Lambda = \frac{\ddot{Y}}{Y} + m^2 \quad (27)$$

$$8\pi(\rho_m + \rho_\Lambda) = \frac{\dot{Y}\dot{Z}}{YZ} - m^2 \quad (28)$$

Thus, we have four non-linear differential equations with five unknowns, namely, *Y, Z,* p_Λ, ρ_m, ρ_Λ. We evaluate these unknowns in the next section.

5. Construction of Cosmological model

To obtain the exact solutions in order to construct a new cosmological model, we consider the exponential law of volumetric expansion of the universe as

$$V = \alpha e^{\beta t} \quad (29)$$

where α and β are constants.

By definition, the spatial volume *(V)* is obtained as

$$V = kYZ \quad (30)$$

From (26), (27), (29), and (30) we obtain the metric coefficients *X, Y,* and *Z* as

$$X = k, \quad Y = \frac{\alpha}{D_3} e^{\frac{\alpha\beta^2 t - D_1 e^{-\beta t}}{2\alpha\beta}}, \text{ and } Z = D_2 e^{\frac{\alpha\beta^2 t + D_1 e^{-\beta t}}{2\alpha\beta}} \quad (31)$$

where D_1, D_2 and D_3 are constants.

Substituting the values of *X, Y,* and *Z* from (31) in (11), the space-time filled with DM and HDE within the framework of *f*(T) gravity becomes

$$ds^2 = dt^2 - k^2 dx^2 - e^{2mx}\left(\frac{\alpha^2}{D_3^2} e^{\frac{\alpha\beta^2 t - D_1 e^{-\beta t}}{\alpha\beta}} dy^2 + D_2^2 e^{\frac{\alpha\beta^2 t + D_1 e^{-\beta t}}{\alpha\beta}} dz^2\right). \quad (32)$$

The metric potentials of this model assume constant values at *t* = 0 and do not vanish for any *t*. Hence the model is free from any type of singularities for finite values of *t*.

Now, we define some cosmologically important physical and kinematical parameters.

The average Hubble's parameter *(H)* is given by

$$H = \frac{1}{3}\sum_{i=1}^{3} H_i = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\frac{\dot{V}}{V} \quad (33)$$

where H_1, H_2, H_3 are the directional Hubble parameters.

To determine whether the model approaches isotropy or not, we discuss the mean anisotropy parameter (A_m) given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right)^2 \quad (34)$$

The scalar expansion (θ) and the shear scalar (σ^2) are respectively defined as

$$\theta = u^\alpha_{;\alpha} = 3H, \quad (35)$$

$$\sigma^2 = \frac{3}{2} A_m H^2. \quad (36)$$

The deceleration parameter is defined as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \quad (37)$$

6. Some Physical and Kinematical Parameters

The energy conservation law of matter, $T^\beta_{\alpha;\beta} = 0$ leads to

$$\dot{\rho}_m + 3H\rho_m = 0. \quad (38)$$

From (14) we have obtained the Torsion scalar as

$$T = -\frac{4m^2\alpha^2 - D_2^2 e^{-2\beta t} + \alpha^2\beta^2}{2\alpha^2}. \quad (39)$$

which is time-dependent.

Also, we have determined the average Hubble's parameter (H), the scalar expansion (θ), the mean anisotropy parameter (A_m), the shear scalar (σ^2), and the deceleration parameters (q), respectively, as

$$H = \frac{\beta}{3}, \quad (40)$$

$$\theta = \beta, \quad (41)$$

$$A_m = \frac{\alpha^2\beta^2 + 3D_1^2 e^{-2\beta t}}{2\alpha^2\beta^2}, \quad (42)$$

$$\sigma^2 = \frac{\alpha^2\beta^2 + 3D_1^2 t^{-2\beta t}}{12\alpha^2}, \quad (43)$$

$$q = -1. \quad (44)$$

The spatial volume is constant at *t* = 0 and increases exponentially with time, i.e., the universe starts with some constant volume and expands exponentially with the passage of time. In this expanding universe the mean Hubble's parameter and expansion scalar are constant throughout. The negative sign of the deceleration parameter is indicating the accelerating phase of the universe as expected in the exponential law of volumetric expansion. The values of the anisotropic parameter and shear scalar show that the universe is anisotropic and shear free throughout the evolution. The deceleration parameter (44), and $\frac{dH}{dt} = 0$ shows the largest value of Hubble's parameter and the fastest rate of cosmic expansion as obtained by Chirde and Shekh [68].

From (38) and (40) we obtained the value of matter-energy density as

$$\rho_m = \frac{1}{\alpha e^{\beta t}}. \quad (45)$$

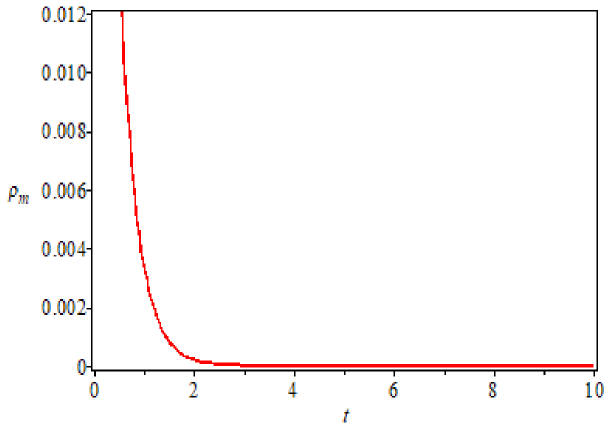

Figure 1. Variation of ρ_m Vs. t for $\alpha = 18,\ \beta = 2.8$

Equation (45) represents the energy density of DM as a function of time *t*. The graphical behavior of the energy density of DM in Figure 1 shows that at an initial epoch when $t = 0$ the matter density is infinite and at a later time it decreases rapidly and assumes a very small constant value (approximately zero) for t >1.8 approx.

Using (45) in (28) we obtain the value of energy density of HDE as

$$\rho_\Lambda = \frac{\left(\beta^2 - 4m^2\right)\alpha^2 - 32\pi\alpha e^{-\beta t} - D_1^2 e^{-2\beta t}}{32\pi\alpha^2}. \tag{46}$$

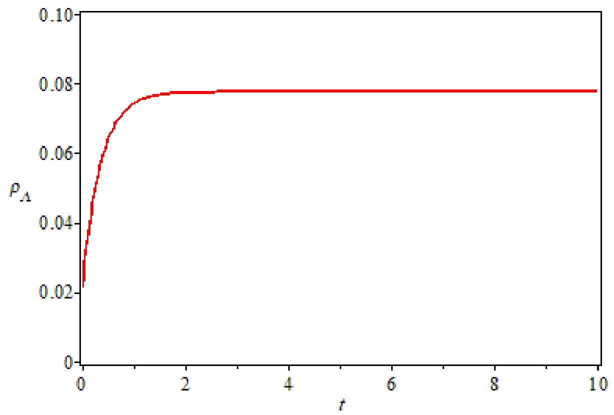

Figure 2. Variation of ρ_Λ Vs. t Vs. t for $\alpha = 18,\ \beta = 2.8$, $m = 0.1,\ D_1 = 5$.

The energy density of HDE given by (46) is a function of cosmic time *t*, and its graphical behavior is shown in Figure 2. From the figure 2, it is observed that the HDE density ρ_Λ is always positive, it is very small at an initial epoch and it increases rapidly and becomes constant for t >1.8 approx.

From (27), the pressure of HDE is obtained as

$$p_\Lambda = -\frac{\left(\beta^2 + 4m^2\right)\alpha^2 + D_1^2 e^{-2\beta t}}{32\pi\alpha^2}. \tag{47}$$

Figure 3 represents the behavior of the HDE pressure versus cosmic time *t*, in which pressure grows in negative in an early epoch of evolution but at a later time it increases rapidly to approach constant value (≈ -0.075) as $t \to \infty$.

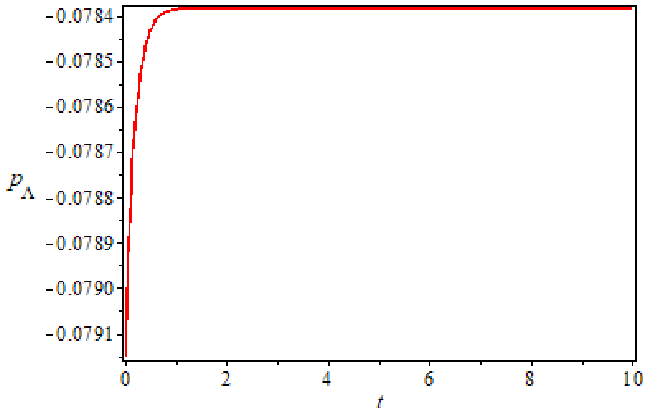

Figure 3. Variation of p_Λ Vs. t for $\alpha = 18,\ \beta = 2.8$, $m = 0.1,\ D_1 = 5$.

The Equation of State (EoS) parameter for HDE is

$$\omega_\Lambda = \frac{\left(\beta^2 + 4m^2\right)\alpha^2 e^{2\beta t} + D_1^2}{\left(4m^2 - \beta^2\right)\alpha^2 e^{2\beta t} + 32\pi\alpha e^{\beta t} + D_1^2}. \tag{48}$$

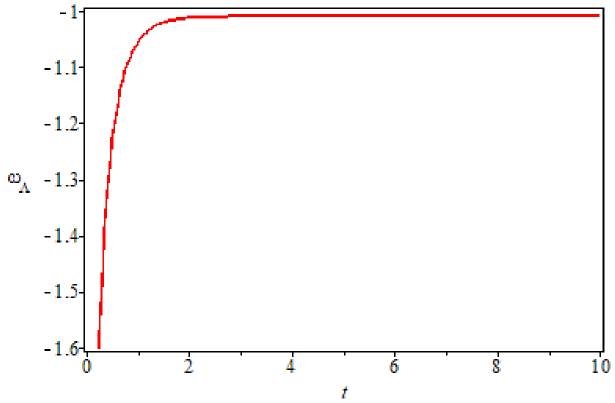

Figure 4. Variation of ω_Λ Vs. t for $\alpha = 18,\ \beta = 2.8$, $m = 0.1,\ D_1 = 5$.

The graphical behavior of the EoS parameter is depicted in Figure 4. The EoS parameter grows from the large negative values and approaches a constant negative value less than -1 for t >1.8 approx. Thus, EoS parameter remains present in the Phantom region i.e., ω_Λ < -1, which is an acceptable value observed by supernova data. As the EoS parameter is time-dependent, the DE influences CMB. According to the type Ia supernovae data, the admissible value of an EoS parameter is in the range -1.67 < ω_Λ < -0.62 [1], while the expected limit for the EoS parameter accompanying the type Ia supernovae data with CMB anisotropy and galactic clustering census is given as $-1.33 < \omega_\Lambda < -0.79$ [61]. It can be observed from the figure that EoS gives an entirely negative epoch which is in good agreement with type Ia supernovae and CMB observational data.

Density parameters, Ω_m for DM and Ω_Λ for HDE, and overall density parameter Ω are obtained as

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{3}{\alpha\beta^2 e^{\beta t}}, \tag{49}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} = \frac{3\left\{\left(\beta^2 - 4m^2\right)\alpha^2 - 32\pi\alpha e^{-\beta t} - D_1^2 e^{-2\beta t}\right\}}{32\pi\alpha^2\beta^2}, \quad (50)$$

$$\Omega = \Omega_m + \Omega_\Lambda = \frac{3\left\{\left(\beta^2 - 4m^2\right)\alpha^2 - D_1^2 e^{-2\beta t}\right\}}{32\pi\alpha^2\beta^2}. \quad (51)$$

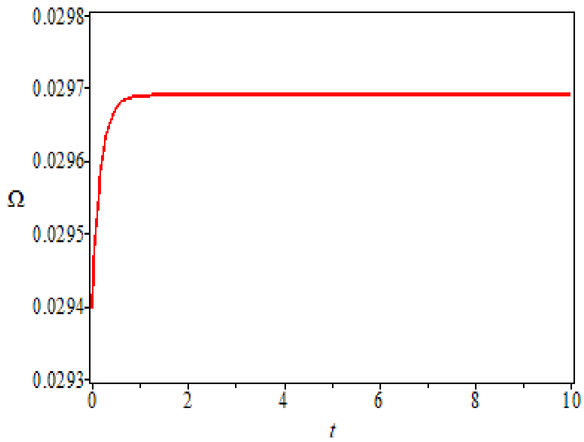

Figure 5. Variation of Ω Vs. t for $\alpha = 18,\ \beta = 2.8,$ $m = 0.1,\ D_1 = 5.$

It can be seen from Figure 5 that the overall density parameter Ω_m grows in positive and approaches the constant value (≈ 0.0297) for t >1.8 approx. which is consistent with the results for a flat universe.

5.1 Stability Factor

For the stability of cosmological solutions obtained, we should examine the nature of quantity advised in cosmological studies. To investigate the physical acceptability and stability of any DE model, the suggested quantity is the squared speed of sound marked by ϑ_s^2. The derived model is stable and unstable according as $\vartheta_s^2 > 0$ and $\vartheta_s^2 < 0$ respectively. As we know that the speed of sound speed is less than the speed of light (c), and for a gravitational unit we hold c = 1, therefore the models in which the squared speed of sound lies between 0 and 1 are physically acceptable.

The squared speed of sound in our exponential volumetric expansion of the model is defined and obtained as

$$\vartheta_s^2 = \frac{\dot{p}_\Lambda}{\dot{\rho}_\Lambda} = \frac{D_1^2}{16\pi\alpha e^{\beta t} + D_1^2} \quad (52)$$

The graphical behavior of the squared speed of sound versus cosmic time t is depicted in Figure 6. It can be seen from the figure that ϑ_s^2 is a decreasing function of cosmic time and for t < 3 it decreases rapidly from infinitely large values whereas when $t \to \infty$ it becomes null and void. The value of ϑ_s^2 seems to lie between 0 and 1 and hence the derived model is always stable. Therefore based on discussion and analysis made here, the constructed model is physically acceptable.

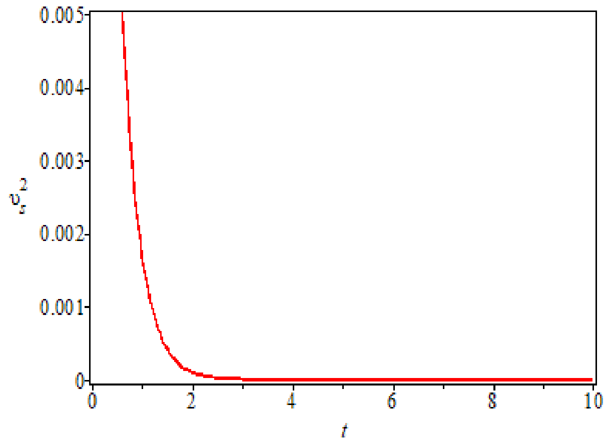

Figure 6. Variation of ϑ_s^2 Vs. t for $\alpha = 18,\ \beta = 2.8,$ $m = 0.1,\ D_2 = 5.$

7. Results and Conclusion

In this paper, we have investigated the spatially homogeneous and anisotropic Bianchi type V cosmological model in the presence of DM and HDE in the framework of *f(T)* gravity. To obtain the deterministic solutions of the constructed model we have considered the exponential volumetric expansion of the universe as $V = \alpha e^{\beta t}$. It is found that the model is free from any type of singularity. The universe starts with some constant volume and expands exponentially as the time increases. In this expanding universe the mean Hubble's parameter and expansion scalar are constant throughout. The constant deceleration parameter $q = -1$ indicates the accelerating expansion of the universe, and $\frac{dH}{dt} = 0$ shows the largest value of Hubble's parameter and hence the fastest rate of cosmic expansion, which is in good agreement with the earlier results. It is observed that the constructed model is shear-free and anisotropic in nature throughout the evolution. It is observed that at an initial epoch when t = 0 the matter density is infinite and at a later time it decreases rapidly and assumes a very small constant value (approximately zero) for t >1.8 approx. The HDE density ρ_Λ is always positive, it is very small at an initial epoch and it increases rapidly and becomes constant for t >1.8 approx. The graphical behavior of EoS parameter shows that it remains present in the phantom region i.e., ω_Λ < -1. According to the type Ia supernovae data, the admissible value of an EoS parameter is in the range -1.67 < ω_Λ < -0.62, while the expected limit for the EoS parameter accompanying the type Ia supernovae data with CMB anisotropy and galactic clustering census is given as –1.33 < ω_Λ < - 0.79. Therefore, the obtained value of EoS parameter is in good agreement with type Ia supernovae and CMB observational data. The overall density parameter Ω grows in positive and approaches the constant value with the elevation of time representing the results for a flat universe and the calculated squared speed of sound lies in 0 to 1 indicating the constructed model is always stable and physically acceptable throughout the evolution.

Conflict of Interest

Authors declare that they do not have any conflict of interest.

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