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Dynamics of Coupled Circle Map on Diffusion Limited Aggregate

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Abstract. Complex networks and d-dimensional Euclidean lattices have both been researched using coupled map lattices. Additionally, it has been examined on the deterministic fractal known as the Sierpinski Gasket. In this work, we investigate the coupled map lattice on a random fractal called diffusion limited aggregate (DLA). We create a map and examine it from the perspective of the circle map. In the event of a DLA, a site's neighbors may number one to four. We examine the scenario in which the total weight does not stay constant. In this regard, we plot bifurcation diagrams.

Keywords: Coupled Map Lattice, Circle Map, Diffusion limited Aggregate

INTRODUCTION

Dynamics-displaying fractals on Coupled Map lattices

A system that has been extensively investigated is coupled map lattices on Euclidean lattices in d-dimensions. In this regard, the most researched maps are circular, tent, and logistical maps. Few studies have been done on the dynamics of fractals. Fractal connectivity scales with distance.

In the case of connected map lattices, such systems exhibit a transition from spatial order to spatially uniform or chaotic states when coupling is changed. Based on simulations of neural networks, coupled oscillators, and coupled maps, nodes are divided into regions of fixed point, chaotic, and oscillating regions.

Network connectivity has an impact on how activities are divided inside networks. We can achieve these partially arrested states in what are known as chimaera states. This article here examines a fractal model known as the diffusion limited aggregate (DLA). Coupled map lattices are dynamical networks that act like complicated models and are spatially homogeneous and computationally feasible. Things having fractal architectures exhibit exciting physical phenomena [3]. In CMLs, coupling is diffusively discrete. Similar to the logistic map, the circle map is a chaotic map. Similar to the dynamics of neurons, chaotic, oscillatory, and fixed-point behavior can be seen. Each of these many dynamical kinds is dependent upon the type of coding and the applied stimulus [4].

If systems exhibit statistical symmetry, long-range interactions, and are probabilistic in nature, they can have chaotic temporal states and long-range spatial order with temporal disorder. We can learn about the stability of randomly connected elements from the Wigner-May theorem. The instabilities to a spatially uniform state vary, and the eigen-values of fractals exhibit intriguing structure [5].

Diffusion Limited Aggregate (DLA)

Written and Sander [1] generated a metal-particle aggregation process model whose correlations were measured. They concluded that, like metal aggregates, the density correlations in the model aggregates decrease with distance along a fractional power law. The metal aggregates' radius of gyration follows a power law pattern.

The DLA model is based on the Eden model, in which randomly added particles are introduced to sites next to occupied sites one at a time. However, Written and Sander discovered that the fractional power law of distance was how the metal aggregates slid off. The irreversible growth process is the source of these relationships. Similarities between the DLA model and the discrete Langer-Krumbhaar model of dendritic development are found [6].

COUPLED MAPS ON DLA

Initially, we start with a seed particle at the lattice origin. Next, we introduce a second particle at a random location, a considerable distance from the origin. Up until it reaches the location next to the seed, the second particle travels at random. After then, this particle joins the cluster. Similar actions are taken by additional particles that are introduced at random times. If a particle crosses the lattice's borders, it is eliminated and a new one is added. Over 10^5 sites are used to recreate the DLA.

We define variable value xi,j(t) to the site (i,j) at time t. The evolution is given by:

$$x_{i,j}(t+1) = (1-\varepsilon)\mathrm{f}\left(x_{i,j}(t)\right) + \frac{\varepsilon}{4}\left(\mathrm{f}\left(x_{i+1,j}(\mathrm{t})\right) + \mathrm{f}\left(x_{i-1,j}(\mathrm{t})\right) + \mathrm{f}\left(x_{i,j+1}(\mathrm{t})\right) + \mathrm{f}\left(x_{i,j-1}(\mathrm{t})\right)\right) \qquad \textbf{(1)}$$

where sites that are not part of the DLA cluster are considered to have contributed nothing and have not changed over time.

Alternatively,

$$x_{i,j}(t+1) = \left(1 - N(i,j)\frac{\varepsilon}{4}\right)\mathrm{f}\left(x_{i,j}(t)\right) + \frac{\varepsilon}{4}\sum_{\eta(i,j)}\mathrm{f}\left(x_{\eta(i,j)}(t)\right) \qquad \textbf{(2)}$$

where N(i,j) is the total number of DLA cluster neighbors for site (i,j). Sites that are not part of the DLA cluster are not taken into account and are not thought to have changed over time.

In order to clarify the distinction, we consider the scenario in which site (i,j) on the cluster has just two neighbors: (i+1,j) and (i-1,j). By (1), the evolution will be:

$$x_{i,j}(t+1) = (1-\varepsilon)\,\mathrm{f}\left(x_{i,j}(t)\right) + \frac{\varepsilon}{4}\left(\mathrm{f}\left(x_{i+1,j}(\mathrm{t})\right) + \mathrm{f}\left(x_{i,j-1}(\mathrm{t})\right)\right) \qquad \textbf{(3)}$$

And that according to (2) will be

$$x_{i,j}(t+1) = \left(1 - \frac{\varepsilon}{2}\right)\mathrm{f}\left(x_{i,j}(t)\right) + \frac{\varepsilon}{4}\left(\mathrm{f}\left(x_{i+1,j}(\mathrm{t})\right) + \mathrm{f}\left(x_{i,j-1}(\mathrm{t})\right)\right) \qquad \textbf{(4)}$$

The total of the weights is not conserved in rule (1), but it is in rule (2). In rule (1), the evolution of a given site is dependent on the number of neighbors; in rule (2), this dependency is absent. A typical DLA cluster produced by the aforementioned technique is displayed in Fig. 1. We examine the coupled circle map's dynamics on the DLA.

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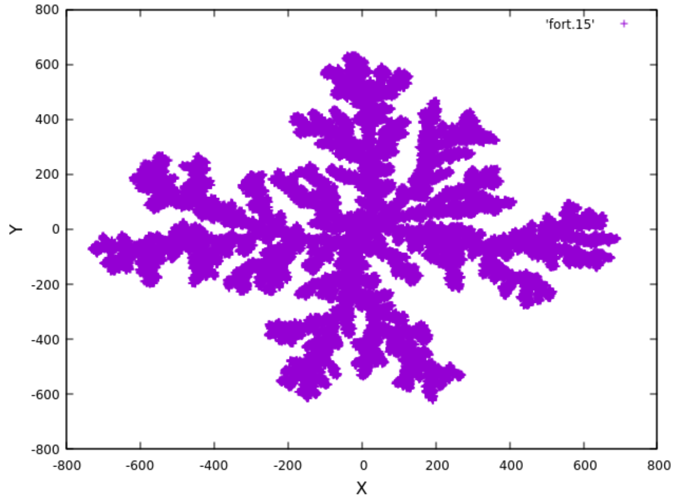

Figure 1: A typical DLA cluster

Circle map

The circle map is a one-dimensional map which maps a circle onto itself, where θ_{n+1} is computed *mod 1* and K is a constant. Note that the circle map has two parameters Ω and K. Ω can be interpreted as an externally applied frequency, and K as a strength of nonlinearity. The circle map exhibits very unexpected behavior as a function of parameters, as illustrated below [7].

$$\theta_{n+1} = \theta_n + \Omega - \frac{K}{2\pi} \sin(2\pi\theta_n),$$

The circle map coupled on a DLA with one, two, three, and four neighbours is now plotted using bifurcation diagrams. Plotting the bifurcation diagrams against the control parameter ε, which ranges from 0 to 1, is done for two-dimensional sites.

Bifurcation Diagrams for Circle Map

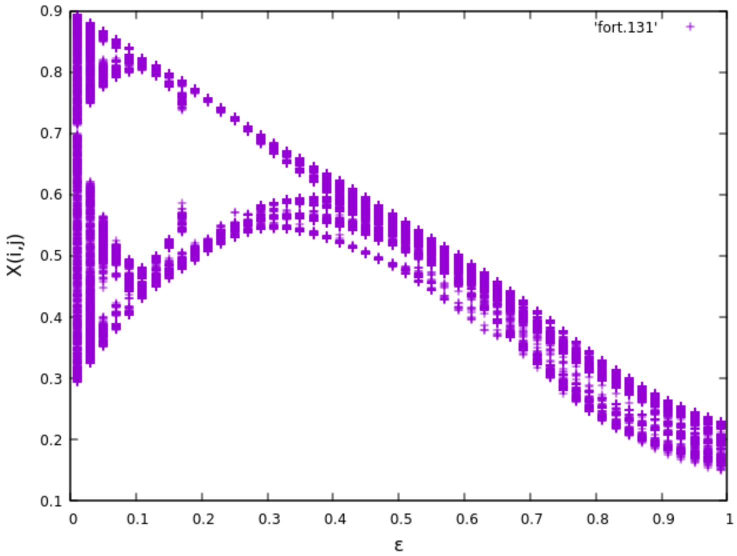

Figure 2: Bifurcation diagram for the non-conserved case for sites with one neighbor.

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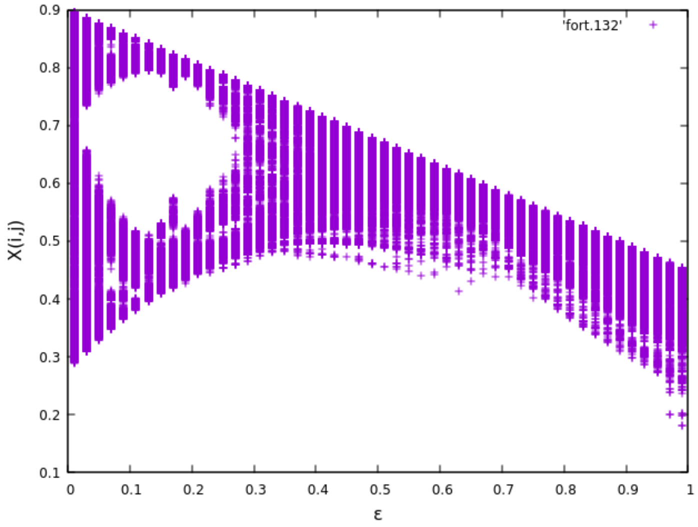

Figure 3: Bifurcation diagram for the non-conserved case for sites with two neighbors.

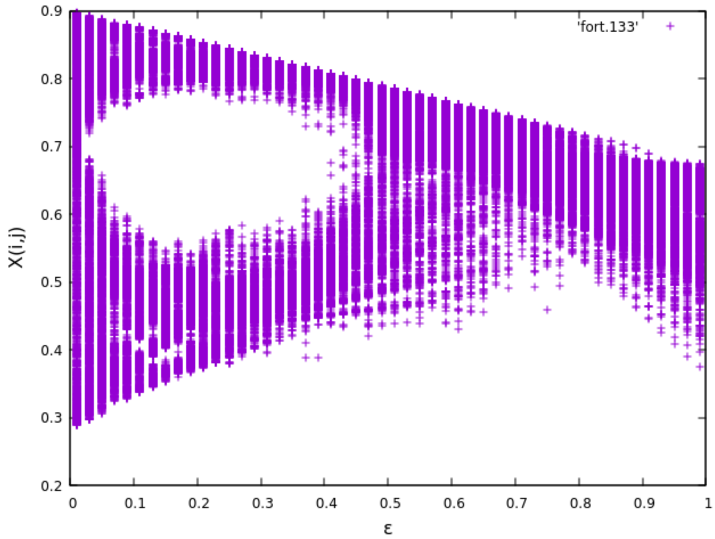

Figure 4: Bifurcation diagram for the non-conserved case for sites with three neighbors.

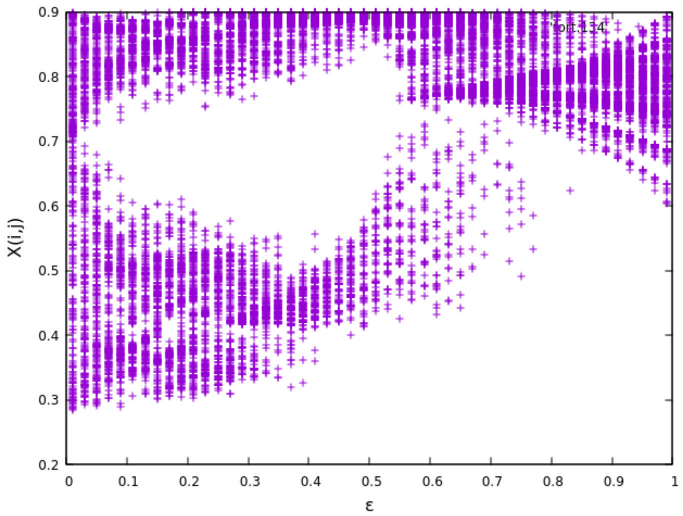

Figure 5: Bifurcation diagram for the non-conserved case for sites with four neighbors.

Conclusion

Since DLA is a well-studied model for random fractals, we examine its dynamics. On this random fractal, we examine coupled circle maps. It is discovered that the band attractor differs in each scenario, whether there are one, two, three, or four neighbours. Although there are no periodic windows, the regions exhibit band periodicity. Graphs of this kind appear to be prevalent in all non-conserved scenarios.

References

1. T. A. Witten Jr and L.M. Sander, Phys. Rev. Lett., **47**(19), 1400 (1981).
2. A. Mansouri and X. Wang, Information Sciences, 2020 – Elsevier
3. B. B. Mandelbrot, The Fractal Geometry of Nature (Freeman, New York, 1982)
4. K. Kaneko, Physica (Amsterdam) **41D**, 137 (1990).
5. M.G. Cosenza and R. Kapral, Phys. Rev. A **46,** 1850 (1992).
6. J.S. Langer and H. Müller-Krumbhaar,ActaMetallurgica**26** (11), 1681-1687 (1978).
7. https://mathworld.wolfram.com/CircleMap.html