Classroom

In this section of *Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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Foucault's Pendulum Exploration Using MAPLE18*

In this article, we develop the traditional differential equation for Foucault's pendulum from physical situation and solve it from standard form. The sublimation of boundary condition eliminates the constants and choice of the local parameters (latitude, pendulum specifications) offers an equation that can be used for a plot followed by animation using MAPLE. The fundamental conceptual components involved in preparing differential equation viz; (i) rotating coordinate system, (ii) rotation of the plane of oscillation and its dependence on the latitude, (iii) effective gravity with latitude, etc., are discussed in detail. The accurate calculations offer quantities up to the sixth decimal point which are used for plotting and animation. This study offers a hands-on experience. Present article offers a know-how to devise a Foucault's pendulum just by plugging in the latitude of reader's choice. Students can develop a miniature working model/project of the pendulum.

Keywords
Foucault's pendulum, rotational motion, latitude, gravity with latitude, differential equation, Coriolis force, centrifugal force, rotational dynamics, MAPLE.

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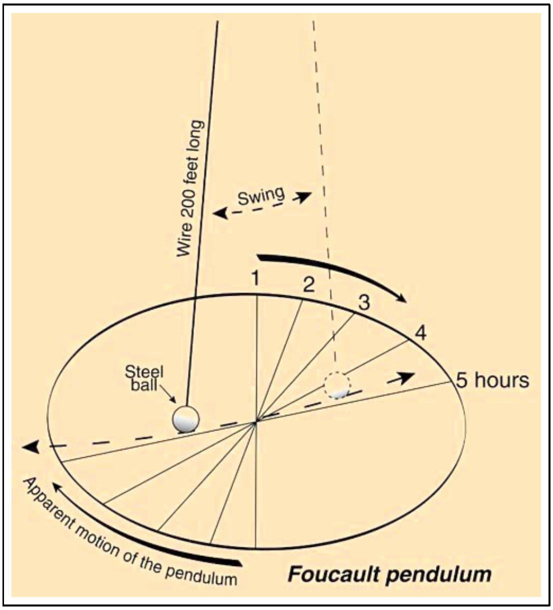

Figure 1. Schematic representation of Foucault's pendulum swinging.
Courtesy: quora.com

Leon Foucault
(1819–1868)
Courtesy: eduspb.com

1. Introduction

Understanding the pendulum is one of the milestones in the development of physics. In 1851, Leon Foucault, conceived a simple devise that demonstrates the spin motion of Earth. This device was tagged with his name Foucault's pendulum (FP) after its public exhibition at the Meridian of Paris observatory. He then used a brass coated lead bob of 28 kg suspended from the roof with a 67 m long wire hooked in the dome of the Panthéon. For the latitude of Paris, the plane of the pendulum's swing rotated clockwise approximately 11.3^{o} per hour, making a full circle in approximately 31.8 hours.

The schematic representation of the bob-traced trajectory of the Foucault's pendulum is depicted in *Figure* 1. A second temporary installation was made for the 50th anniversary in 1902 [1]. During the reconstruction of the museum (1990), the original pendulum was temporarily displayed at the Panthéon (1995) [2]. In 2010, the cable suspending the bob snapped [3, 4].

When FP is suspended at the North or South Pole, the plane of oscillation of the pendulum remains fixed relative to the distant mass of the universe while the Earth rotates underneath it. Therefore, with reference to Earth, the plane of oscillation of the pendulum at the North Pole undergoes a full clockwise rotation during one day; a pendulum at the South Pole rotates counter-clockwise. On the contrary, at the Equator, its plane of oscillation remains fixed to the earth. At intermediate latitudes, the plane of oscillation rotates as a sinusoidal function of the latitude. The involved precisional motion is governed by the solution of the differential equation (DE) of the FP. We shall now rush through all the components that are used in building up the DE and its solution – manually and by exploring MAPLE18.

2. Theory: The Differential Equation and Solution

We shall process the basic components to build the partial differential equation (DE) for FP followed by the theoretical solution in terms of parametric equations followed by its plotting and animations.

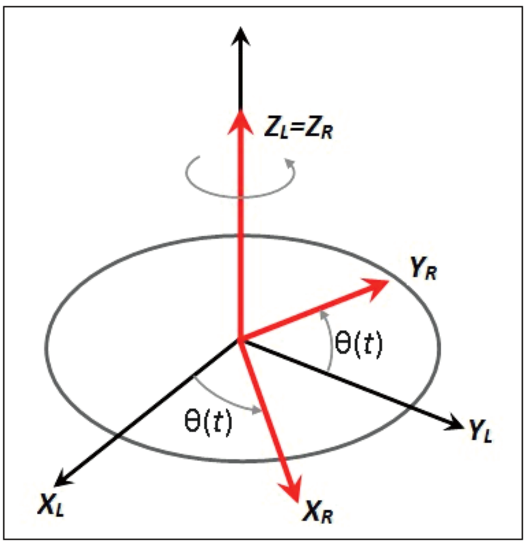

Figure 2. Fixed rectangular coordinate system S, named laboratory coordinate axes X_L, Y_L and Z_L, and rotating coordinate system S' about Z axis denoted by another set of mutually perpendicular coordinate axes X_R, Y_R and Z_R. At time t, the angular description $\theta(t)$ is depicted while Z being the axis of rotation, remains unchanged: $Z_R = Z_L$.

2.1 Basics of Rotating Coordinate System

We follow the equations of circular geometry in which $\overrightarrow{r} = x\hat{i} + y\hat{j} = r\cos\theta\hat{i} + r\sin\theta\hat{j} = r\hat{i'}$. Thus, we have new unit vector in the rotating frame: $\hat{i'} = \cos\theta\hat{i} + \sin\theta\hat{j}$. Its perpendicular vector $\hat{j'}$ can be obtained simply by adding $\pi/2$ in the angle θ. Thus, we have, $\hat{j'} = \cos(\theta + \pi/2)\,\hat{i} + \sin(\theta + \pi/2)\hat{j} = -\sin\theta\hat{i} + \cos\theta\hat{j}$.

We propose to consider two coordinate systems S and S' offering a fixed and a rotated coordinate system at an angle θ respectively. To mark a continuously rotating coordinate system, we prefer this angle θ to be a function of time and the rate of rotation to be accounted by a parameter ω (often referred as the angular frequency) as $\frac{d}{dt}\theta(t) = \omega$ with obvious angular period 2π. *Figure* 2 represents two such coordinate systems at some time t. The relation describing the position and velocity in coordinate systems S and S' are:

$$\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k},$$
$$\overrightarrow{r'} = x'\hat{i'} + y'\hat{j'} + z'\hat{k'}\ .$$

The position vector physically remains the same though observers in two different frames S and S' designate them as $\overrightarrow{r}$ and $\overrightarrow{r'}$ respectively. As this entity 'position vector' is the same, we mathematically quote this in the form of (1):

$$\overrightarrow{r} = \overrightarrow{r'}\ . \tag{1}$$

The position vector $\overrightarrow{r} = r(\cos\theta\hat{i} + \sin\theta\hat{j}) = r\hat{a}_r$, for polar unit vector $\hat{a}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$, and its time derivative by chain rule $\frac{d\overrightarrow{r}}{dt} = \frac{d\overrightarrow{r}}{d\theta}\frac{d\theta}{dt}$, at $r = const$. For $\frac{d\theta(t)}{dt} = \omega$ and $\frac{d\overrightarrow{r}}{d\theta} = r\frac{d}{d\theta}\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) = r(-\sin\theta\hat{i} + \cos\theta\hat{j}) = r(\hat{a}_\theta)$ for the another polar unit vector $\hat{a}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$. Thus, we have, $\frac{d\overrightarrow{r}}{dt} = \omega r\hat{a}_\theta$.

Here, we consider that S' coordinate system is rotating and hence we have non-vanishing $\frac{d\hat{i'}}{dt}, \frac{d\hat{j'}}{dt}$ unlike vanishing $\frac{d\hat{i}}{dt}, \frac{d\hat{j}}{dt}$. Thus, differentiating the position vector with respect to time t depicted in (1) to obtain velocity:

$$\frac{d\overrightarrow{r}}{dt} = \frac{d\overrightarrow{r'}}{dt}\ . \tag{2}$$

$$\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \frac{d(x'\hat{i'})}{dt} + \frac{d(y'\hat{j'})}{dt}\ ,$$

$$\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \frac{d(x')}{dt}\hat{i'} + x'\frac{d(\hat{i'})}{dt} + \frac{d(y')}{dt}\hat{j'} + y'\frac{d(\hat{j'})}{dt}\ ,$$

Substituting: $\frac{dx'}{dt} = v'_x$, $\frac{dy'}{dt} = v'_y$, $\frac{dx}{dt} = v_x$, $\frac{dy}{dt} = v_y$, $\frac{d}{dt}\hat{i}' = \omega\hat{j}'$, $\frac{d}{dt}\hat{j}' = -\omega\hat{i}'$, $\frac{d}{dt}\hat{i} = 0$ and $\frac{d}{dt}\hat{j} = 0$,

$$v_x\hat{i} + v_y\hat{j} = v'_x\hat{i}' + x'\omega\hat{j}' + v'_y\hat{j}' - y'\omega\hat{i}' \ .$$

For $v_x\hat{i} + v_y\hat{j} = \overrightarrow{v}, v'_x\hat{i}' + v'_y\hat{j}' = \overrightarrow{v'}$ and $\overrightarrow{\omega} = \omega\hat{k}$, we get the emperical relationship between velocity in two frames from (2) as:

$$\overrightarrow{v} = \overrightarrow{v'} + \overrightarrow{\omega} \times \overrightarrow{r} \ . \tag{3}$$

Above equation (3) in terms of displacement vector can be expressed in the form of a differential operator as following:

$$\frac{d\overrightarrow{r}}{dt}|_S = \frac{d\overrightarrow{r'}}{dt}|_{S'} + \overrightarrow{\omega} \times \overrightarrow{r} \tag{4}$$

Thus, using (4), we can set-up the differential operator which caters us to find the vector in frame S if we know that vector in frame S'. This differential operator can be expressed as: $\frac{d}{dt}|_S \longrightarrow \frac{d}{dt}|_{S'} + \overrightarrow{\omega}\times$. The differential operator will enable us calculate the velocity relationship if we know the position vector and its repeated use enable us to find relation acceleration stating acceleration in two frames – S and S'.

MATRIX lines of latitude and longitude come together to form a matrix/grid. It will allow you to pinpoint your location with a high degree of accuracy. Latitude is the angular distance measured north and south of the Equator. Latitude on the Equator is 0°, 90° at the North Pole, and −90° at the South Pole.

$$\left(\frac{d}{dt}\frac{d}{dt}\right)_S \longrightarrow \left(\left(\frac{d}{dt}\right)_{S'} + \overrightarrow{\omega}\times\right)\left(\left(\frac{d}{dt}\right)_{S'} + \overrightarrow{\omega}\times\right),$$

which on simplification offers a form as following when operated on position vector:

$$\left(\frac{d^2}{dt^2}\right)_S \longrightarrow \left(\frac{d^2}{dt^2}\right)_{S'} + \frac{d\overrightarrow{\omega}}{dt} \times \overrightarrow{r} + 2\overrightarrow{\omega} \times \frac{d\overrightarrow{r}}{dt} + \overrightarrow{\omega} \times \left(\overrightarrow{\omega} \times \overrightarrow{r}\right) \ .$$

Now, if the observer is in the rotating frame (say on the earth) and would like to know the quantity, say acceleration, in the fixed frame (Sun) then the equations are required to be re-arranged and we shall prefer to incorporate the term angular acceleration $\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t}$ in the last relation, we get:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right)_{S'} \longrightarrow \left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right)_{S} - \vec{\alpha} \times \vec{r} - 2\vec{\omega} \times \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} - \vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right) . \quad (5)$$

If we multiply the above relation by mass m on both sides, we have the relationship:

$$m\vec{a'} = m\vec{a} - m\vec{\alpha} \times \vec{r} - 2m\vec{\omega} \times \vec{v} - m\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right) .$$

Thus, the force in rotating frame S' (Earth) is termed as;

$$\vec{F'} = \vec{F} - \vec{F}_{\mathrm{EULER}} - \vec{F}_{\mathrm{CORIOLIS}} - \vec{F}_{\mathrm{CENTRIFUGAL}} . \quad (6)$$

In (6), we have $\vec{F'} = m\vec{a'}$ representing the force in rotating frame (say observer on the Earth), $\vec{F} = m\vec{a}$ represents force in fixed frame (observer steady with respect to Earth, say observer on Sun), $\vec{F}_{\mathrm{EULER}} = -m\vec{\alpha} \times \vec{r}$ for $\vec{\alpha}$ being angular acceleration, $\vec{F}_{\mathrm{CORIOLIS}} = -2m\vec{\omega} \times \vec{v}$ is the Coriolis force, (refer *Figure* 3)

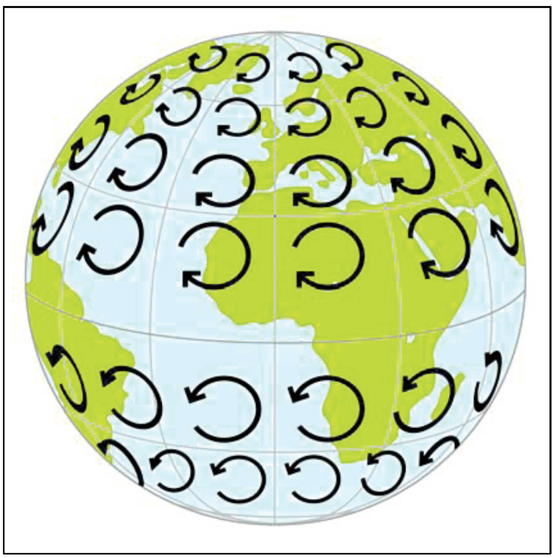

Figure 3. The Coriolis force causing rotation of the plane of oscillation of Foucault's pendulum in clockwise and counter-clockwise direction in northern and southern hemisphere marked by the Equator. Courtesy: neatorama.com

$\vec{F}_{\text{CENTRIFUGAL}} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})$ is the centrifugal force. This Coriolis force is capable of rotating the plane of oscillation of FP in the clockwise direction in the northern hemisphere while in the counter-clockwise direction in the southern hemisphere elaborated in the next article.

2.2 Rotation of the Plane of Oscillation of FP

Due to the spin motion of Earth, the plane of oscillation of FP rotates. On the North Pole, the plane of oscillation describes a complete rotation in 24 hrs. As one travels towards the Equator, its period of rotation decreases, and at the Equator, the rotation of the plane ceases which turns counter-clockwise as one travels towards the South Pole to regain its period to 24 hrs. Mike Town and John Bird of University of Washington carried out experiments to physically verify the period of rotation of FP which con-

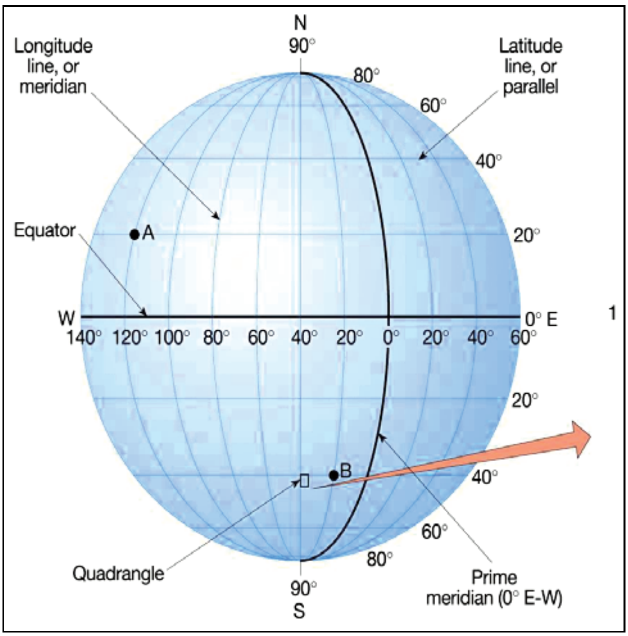

Figure 4. Depiction of the Poles, Equator, Latitudes and Longitudes.
Source: http://bookmarkurl.info/images/what-are-latitude-and-longitude/what-are-latitude-and-longitude-5.jpg

cluded the same. A similar reporting has also been made earlier [5]. Now, we shall find the functional variation of this rate of rotation of the plane of oscillation (angular frequency $\omega = 360/24 = 15^{o}$/hr) of FP as a function of the latitude. Any point on spherical Earth can be designated by latitude and longitude (angle) as shown in *Figure* 4.

Now, consider two points in close proximity, say, P_1 and P_2 on the surface of Earth which we shall consider spherical in shape and spinning about an axis passing through the north–south pole. Let the angular velocity of Earth be $\overrightarrow{\omega}$ that will cause the rotation of point P_2 about P_1 with velocity $\overrightarrow{v}$. The basic rotational motion offers relationship:

$$\overrightarrow{v_1} = \overrightarrow{\omega} \times \overrightarrow{r_1},$$
$$\overrightarrow{v_2} = \overrightarrow{\omega} \times \overrightarrow{r_2}. \tag{7}$$

Here r is the radius of spherical Earth forming one of the spherical coordinates. The vector $\overrightarrow{r}$ of the point P_1 describes latitude θ_1 while the point P_2 describes latitude θ_2. Their distances from axis of rotation are $r_1 = r\cos\theta_1, r_2 = r\cos\theta_2$ for Earth revolving about an axis passing through north–south pole.

Thus, it is obvious that $\overrightarrow{\omega}$ is normal to $\overrightarrow{r_1}$ and $\overrightarrow{r_2}$, and hence we can simplify the angular velocity of P_1 about P_2 and P_2 about P_1 are $v_1 = \omega r_1$ and $v_2 = \omega r_2$ respectively. Now, the difference in

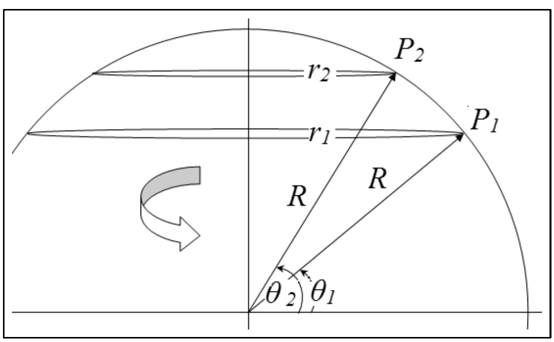

Figure 5. Rotation of a point P_1 about P_2 on the spinning Earth.

Place	θ	ω_P	T_P
P	degree	{μrad/s}	hours
Paris	48.86	54.77	31.86
Shrinagar	34.083	40.75	42.82
New Delhi	28.644	34.86	50.06
Nagpur	21.146	26.23	66.52
Bangalore	12.972	16.32	106.91
Kanyakumari	08.088	10.23	170.57
Sydney	-33.865	-40.52	-43.06

Table 1. Comparative data for latitude θ, angular frequency ω_P, and period of rotation of the plane of oscillation T_P for the place P.

the velocity can be expressed as $v_{\text{dif}} = v_1 - v_2 = \omega(r_1 - r_2)$. Note that here r_1, r_2 are the radius of the points P_1, P_2 from the center of earth which is origin of coordinate system (see *Figure* 5). This quantity can be expressed from the geometry as $r_{\text{dif}} = r_1 - r_2 = R\sin(\theta_2 - \theta_1)$. Thus, we have:

$$|\overrightarrow{\omega_{P1}}| = \frac{v_{\text{dif}}}{r_{\text{dif}}} = \frac{\omega(r_1 - r_2)}{r\sin(\theta_2 - \theta_1)} .$$

From the geometry, we have $r_1 = R\cos(\theta_1)$, $r_2 = R\cos(\theta_2)$ which on substitution in the above equation leads to:

$$|\overrightarrow{\omega_{P1}}| = \frac{\omega(\cos(\theta_1) - \cos(\theta_2))}{\sin(\theta_1 - \theta_2)} .$$

Here, substituting $\theta_2 = \theta_1 + \Delta\theta$ in the numerator and $\theta_2 - \theta_1 = \Delta\theta$ in the denominator, we get:

$$|\overrightarrow{\omega_{P1}}| = \frac{\omega(\cos(\theta_1) - \cos(\theta_1 + \Delta\theta))}{\sin(\Delta\theta)} .$$

For the limiting condition of $\Delta\theta \to 0$, we have P_1, P_2 close to each other, the above equation reduces to (8) using basic trigonometric identities:

$$\lim_{\Delta\theta \to 0} |\overrightarrow{\omega_{P1}}| = |\omega| \sin\theta_1 . \tag{8}$$

Reader may calculate this data for his latitude. However, a set of this data for sample places is depicted in *Table* 1.

Note that, on substitution, the angular velocity of Earth is $\omega = \frac{d\theta}{dt} = \frac{2\pi}{24 \times 60 \times 60} = 72.72 \times 10^{-6}$ rad/sec or (15^{o}/hr). For latitude of Paris (P) $\omega_P = 54.77 \times 10^{-6}$ rad/sec=11.29^{o}/ hr, and the period $T_P = \frac{2\pi}{\omega_P} = 31.86$ hrs.

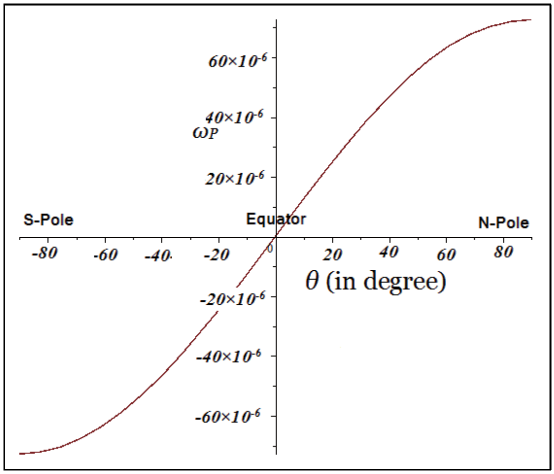

Figure 6. Variation of angular frequency ω_P with latitude.

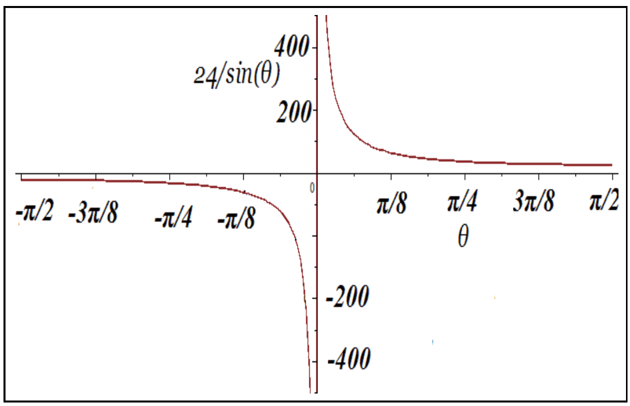

Figure 7. Variation of period of rotation of FP with latitude.

At the North Pole, the time period of rotation of the plane of oscillation of Foucault's pendulum is $T = 24$ hrs which increases as one travels to the Equator ceasing rotation at Equator (see *Figure* 6). Further, as one proceeds towards South Pole from the Equator, time period regains periodicity which goes on decreasing until 24 hrs at South Pole. *Figure* 7 shows variation of the period of rotation of FP as one rolls from the north to south pole. The meaning of negative sign appearing with the time period T is attributed to counter clockwise rotation.

2.3 Gravity with Latitude

For practical purposes, we consider acceleration due to the gravity g = 9.80655 m/s^2. However, for precise calculations, we shall consider more accurate value of gravity that depends upon the latitude and elevation. At the Pole, gravitational pull is maximum and its accurate value is g = 9.787458554 m/s^2 and is known as the normal equatorial value. Due to the spin motion of Earth, the centrifugal force $m\omega^2 r$ depends upon the latitude as the axis of rotation (passing through N–S) alters the centrifugal force as $r = R\cos\theta$ is the radius of rotation. Thus, we have two forces (i) F_{gravity} represented by $\overrightarrow{PO}$ and (ii) $F_{\text{centrifugal}}$ represented by $\overrightarrow{PA} = \overrightarrow{OB}$. The vector addition rule offers us:

$$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB}\,,$$

$$|\overrightarrow{PB}|^2 = |\overrightarrow{PO}|^2 + |\overrightarrow{OB}|^2 + 2|\overrightarrow{PO}||\overrightarrow{OB}|\cos(\pi-\theta)\ .$$

Note that the angle subtended by A and P at O is $\pi - \theta$ though we have written equivalent of $\overrightarrow{PA}$ as $\overrightarrow{OB}$ (refer *Figure* 8). The expression can be rewritten as:

$$(mg_{\text{eff}})^2 = (mg)^2 + \left(m\omega^2 R\cos\theta\right)^2 + 2\,(mg)\left(m\omega^2 R\cos\theta\right)\cos(\pi-\theta)\ .$$

Simplifying the above expression, ignoring the terms pertaining

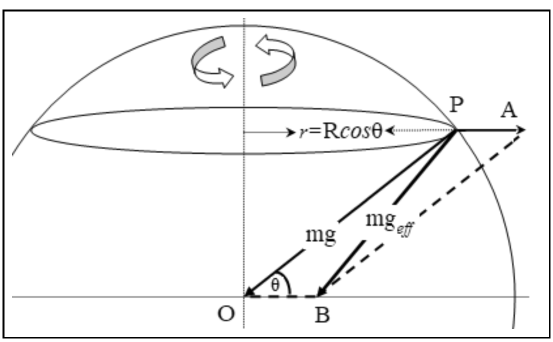

Figure 8. Due to the spin motion, the gravitational pull mg is deferred by centrifugal force $m\omega^2 R\cos\theta$ directed along AP and yields a resultant mg' along PB.

Table 2. Comparative data for latitude θ and effective gravity for that place P using the relationship $g_{\text{eff}} = g - R\,(\omega\cos\theta)^2$.

Place	θ	g_{eff}
P	degree	$\{\mathrm{m/s^2}\}$
Paris	48.86	9.80495
Shrinagar	34.083	9.79638
New Delhi	28.644	9.79353
Nagpur	21.146	9.79015
Bangalore	12.972	9.78745
Kanyakumari	08.088	9.78642
Sydney	-33.865	9.79626

to ω^4 (as ω is small in rad/sec), we get expression for effective gravity as a function of latitude θ:

$$g_{\text{eff}} = g\sqrt{\left(1 - \frac{2R\,(\omega\cos\theta)^2}{g}\right)}.$$

Thus here, we have $b = i\omega\sin\theta = i\omega_P$ which is an imaginary quantity and its derived parameter will be $\alpha = \sqrt{b^2 - \Omega^2} = \sqrt{-\omega_P^2 - \Omega^2} = i\beta$ for $\beta = \sqrt{(\omega_P)^2 + \Omega^2}$. Thus, the ratio of two imaginary quantities $\dfrac{b}{\alpha} = \dfrac{\omega_P}{\beta}$ makes a real number.

The binomial expansion ignoring powers higher than ω^2, we get:

$$g_{\text{eff}} = g - R\,(\omega\cos\theta)^2\ , \tag{9}$$

here, g refers to the gravity at Poles, g_{eff} is the gravity at the point where latitude is θ and R refers to the radius of Earth 6400 km. *Table* 2 depicts data for few places we are considering for discussion using (9). Note that on the Equator, effective gravity is minimum while it symmetrically increases as one travel towards the Poles. *Figure* 9 presents the graphical variation of the (9) over a tour from the South to North Pole via the Equator.

2.4 Pair of Differential Equations Representing the Motion of FP

The DE for FP obviously offers Newtonian force and Coriolis force proportional to the displacement. As the plane of oscillation is rotating due to Coriolis force, the parametric displacement $x(t), y(t)$ are cross connected with the velocity components $\dfrac{\mathrm{d}y(t)}{\mathrm{d}t}, \dfrac{\mathrm{d}x(t)}{\mathrm{d}t}$ respectively through $\sin(\theta)$. The XY plane is tangential to the point where the latitude is being discussed. Thus, the pair of DE takes the form as depicted in (10) and (11):

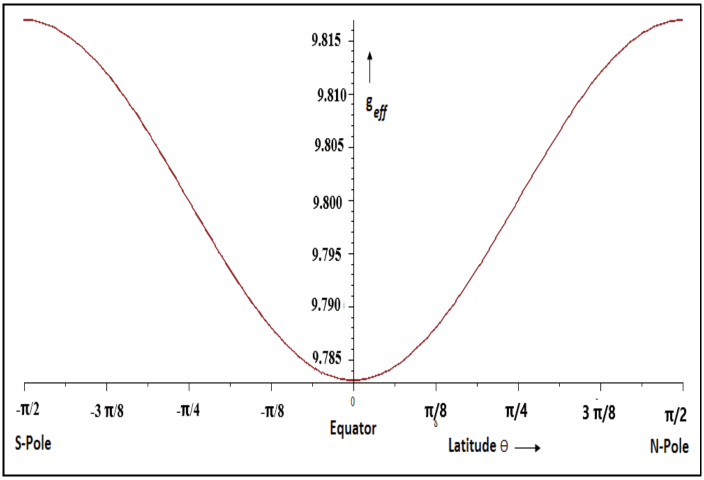

Figure 9. Variation of gravity g with latitude θ.

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) - 2m\omega\sin\theta\frac{\mathrm{d}}{\mathrm{d}t}y(t) = -\frac{mg}{L}x(t)\,. \tag{10}$$

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + 2m\omega\sin\theta\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -\frac{mg}{L}x(t)\,. \tag{11}$$

For compact representation of (10) and (11), we shall explore short-hand notations/substitutions; $\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) = \ddot{x}$, $\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) = \ddot{y}$, $\frac{\mathrm{d}}{\mathrm{d}t}x(t) = \dot{x}$, $\frac{\mathrm{d}}{\mathrm{d}t}y(t) = \dot{y}$, $\frac{g}{L} = \Omega^2, z(t) = x(t) + iy(t), \dot{z} = \dot{x} + i\dot{y}$ and $\ddot{z} = \ddot{x} + i\ddot{y}$. Thus, we have a pair of DE for FP as:

$$\ddot{x} = -\Omega^2 x + 2\omega\sin\theta\dot{y},$$
$$\ddot{y} = -\Omega^2 y + 2\omega\sin\theta\dot{x}\,. \tag{12}$$

Performing 1st + i × 2nd on the pairs in (12) and exercising little simplification for $b = i\omega_P = i\omega\sin\theta$, we get a single DE for complex variable as z:

$$\ddot{z} + 2b\dot{z} + \Omega^2 z = 0\,. \tag{13}$$

For instance, at zero mile of Nagpur, we have, (i) $\theta = 21.146633^{\circ}$, (ii) $\omega = 72.722 \times 10^{-6}$ rad/s, (iii) say $L = 67$ meters and (iv) $g = 9.790158554$ m/s^2, and the derived quantities (v) $\omega_P = 26.234918 \times 10^{-6}$, (vi) $x_o = 6.7$, (vii) effective gravity $g = 9.790158554$, (viii) $\Omega = \sqrt{g_{\text{eff}}/L} = 0.3822587730$, $\beta = 0.3822587739$, $\frac{b}{\alpha} = \frac{\omega_P}{\beta} = 68.631 \times 10^{-6}$.

Now, we have (13) which is a second order ordinary DE for complex variable. The solution of standard DE (13) takes a form as:

$$z = \exp(\lambda t)\,. \tag{14}$$

The parameter λ is to be found and should necessarily satisfy the DE(13) on substitution of its solution, i.e., (14).

$$\exp(\lambda t)\left(\lambda^2 + 2b\lambda + \Omega^2\right) = 0\,.$$

Here, the vanishing of the product of the duo means that either of the duo is vanishing. However, $\exp(\lambda t) \neq 0$, otherwise it will render zero solution which is completely meaningless. Therefore, the second part must vanish, i.e., $\left(\lambda^2 + 2b\lambda + \Omega^2\right) = 0$. This quadratic equation imposes two roots of λ:

$$\lambda = -b \pm \sqrt{b^2 - \Omega^2} = -b \pm \alpha\,. \tag{15}$$

Here, we shall denote two possible solutions found in (15) as $\lambda_1 = -b+\alpha$, and $\lambda_2 = -b-\alpha$ for $\alpha = \sqrt{b^2 - \Omega^2}$, and the unique solution may be devised as being linearly dependent of both. Thus, we have,

$$z(t) = A \exp\left[(-b + \alpha)\right] t + B \exp\left[(-b - \alpha)t\right]\,. \tag{16}$$

Exploring the boundary conditions (BC) to eliminate constants A and B, we have to set up initial conditions as – BC_1:=At at $t = 0$ we set oscillations in the XZ-plane, thus $x(0) = x_o, y(0) = 0 \Rightarrow z(0) = x_o$, BC_2:=At $t = 0$ when we set oscillations in the XZ-plane. Thus $\dot{x}(0) = 0, \dot{y}(0) = 0 \Rightarrow z(0) = 0$. Substituting BC in the solution which is (16), we get;

$$A + B = x_o,\ A(-b + \alpha) + B(-b - \alpha) = 0\,.$$

The above pair of equations can be solved for the pair of constants A and B to obtain these constants:

$$A = \frac{x_o}{2}\left(1 - \frac{b}{\alpha}\right), \ B = \frac{x_o}{2}\left(1 + \frac{b}{\alpha}\right). \tag{17}$$

Thus, substituting A and B from (17) in the solution (16) and simplifying, we get the final solution as (18) of the DE of FP as following:

$$z(t) = \frac{x_o}{2} e^{-bt}\left\{\left(1 + \frac{b}{\alpha}\right)e^{\alpha t} + \left(1 - \frac{b}{\alpha}\right)e^{-\alpha t}\right\},$$

$$z(t) = x_o e^{-bt}\left\{\left(\frac{e^{i\beta t} + e^{-i\beta t}}{2}\right) + i\frac{b}{\alpha}\left(\frac{e^{i\beta t} - e^{-i\beta t}}{2i}\right)\right\}. \tag{18}$$

3. Exploring MAPLE18

Here, the constant b being imaginary, we have the following identity to be substituted in the last equation:

$$e^{-bt} = e^{-i(\omega_P)t} = \cos(\omega_P t) - i\sin(\omega_P t),$$

$$\frac{b}{\alpha} = \frac{\omega_P}{\beta} = \frac{\omega\ sin\theta}{\sqrt{(\omega\sin\theta)^2 + \Omega^2}},$$

$$\frac{e^{i\beta t} + e^{-i\beta t}}{2} = \cos\beta t, \frac{e^{i\beta t} - e^{-i\beta t}}{2i} = \sin\beta t,$$

The complex quantity $z(t)$ in (18) possess a real part $x(t)$ and an imaginary part $y(t)$ which on comparison with the $Re\{RHS(eq\ (18))\}$ and $Im\{RHS(eq\ (18))\}$ with a few bits of simplifications offer us desired solution that we are looking for are the solutions of pair of DE of FP as (10) and (11) are:

$$x(t) = x_o\left\{\cos(\omega_P t)\cos(\beta t) + \frac{\omega_P}{\beta}\sin(\omega_P t)\sin(\beta t)\right\}. \tag{19}$$

$$y(t) = x_o\left\{\frac{\omega_P}{\beta}\cos(\omega_P t)\sin(\beta t) - \sin(\omega_P t)\cos(\beta t)\right\}. \tag{20}$$

Unfolding all the notations which we have inducted for the convenience, we shall now call back the expanded form of the pair of solutions.

$$x(t) = x_o \cos[(\omega \sin\theta)t] \cos\left(t\sqrt{(\omega\sin\theta)^2 + \Omega^2}\right) + \frac{x_o\omega\ sin\theta}{\sqrt{(\omega\sin\theta)^2+\Omega^2}} \sin[(\omega\sin\theta)t] \sin\left(t\sqrt{(\omega\sin\theta)^2+\Omega^2}\right) . \quad (21)$$

$$y(t) = \frac{x_o\omega\ sin\theta}{\sqrt{(\omega\sin\theta)^2+\Omega^2}} \cos[(\omega\sin\theta)t] \sin\left(t\sqrt{(\omega\sin\theta)^2+\Omega^2}\right) - \sin[(\omega\sin\theta)t] \cos\left(t\sqrt{(\omega\sin\theta)^2+\Omega^2}\right) . \quad (22)$$

Now, worldly parameters for the place are to inducted in (21) and (22) where the FP is to be build or theoretically estimated for its parameters, derived parameters and displacement. On submission of these parameters, one obviously desires to plot the instantaneous displacements in terms of $x(t)$, $y(t)$ after substituting them in (21) and (22). *Table* 2 depicts the concerned data for a few places.

MAPLE is a symbolic and numeric computing environment and is also a multi-paradigm programming language. Developed by Maplesoft, it also covers other aspects of technical computing, including visualization, data analysis, matrix computation, and connectivity.

We have displacement along x and y which are functions of time being represented by long equations (24) and (25). Use of MAPLE offers a great ease at handling it once written using the proper syntax. In the next step, we can offer substitutions for the place where we want to install the FP, say, for Nagpur. We offer all substitutions to generate the same equation for numerical values. Note that we are processing calculations for time in minutes (not in seconds) so that the plots for 66.52 hrs should not get masked on time scale. The MAPLE syntax follows:

$restart;$

$with(plots):$

$\phi := evalf(21.146633 * Pi * (1/180)); g := 9.79015;$

$L := 67; x[0] := 6.7; \Omega := evalf(2 * Pi/(24 * 60));$

$\omega := sqrt(g/l); b := I * \Omega * \sin(\phi); abs(b):$

$beta := sqrt(\omega^2 + (\Omega * \sin(\phi))^2):$

$x(t) := 6.9(\cos(\beta t)\cos(|b|t) - (|b|)/(\beta)\sin(\beta t)\sin(|b|t));$

$y(t) := -6.9 * (\cos(\beta * t) * \sin(|b| * t) + (|b|)/(\beta)\sin(\beta * t) * \cos(|b| * t));$

$assign(Soln);$

$evalf([x(t), y(t)])$

Here, MAPLE18 generates output as following.

$$x(t) = 6.9\cos(0.382261t)\cos(0.001574t)$$
$$-0.028413\sin(0.382261t)\sin(0.001574t)$$

$$y(t) = -6.9\cos(0.382261t)\sin(0.001574t)$$
$$-0.028413\sin(0.382261t)\cos(0.001574t)$$

$plot([x(t), y(t)], t = 0...2000,$

$color = [red, blue], thickness = 2)$

$plot([x(t), y(t), t = 0...300])$

generates the real-time parametric plot of $x(t)$ versus $y(t)$ when time in minutes freely run for 5 *hrs* which is displayed in *Figure* 10.

Now, the execution of following simple one line animation command in Maple generates an amazing real-time out put depicted in *Figure* 11. The geometry of the trajectory resembles the work of J Opera [6]. This output is similar to the sand-plot generated by the FP with a pin beneath the bob rolling on the sand as depicted in *Figure* 12.

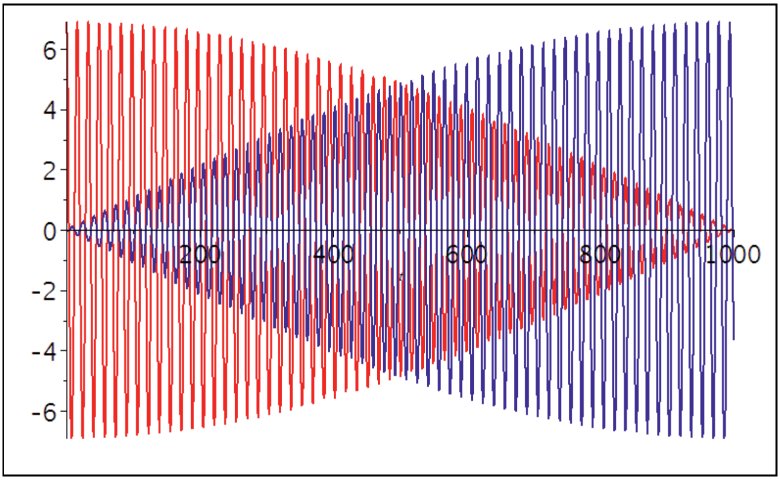

Figure 10. Plot of $x(t)$ and $y(t)$ versus time (in minutes) from (21) (22) for the latitude of Nagpur describing one complete rotation of the plane of oscillation of FP which is described in 66.52 hrs.

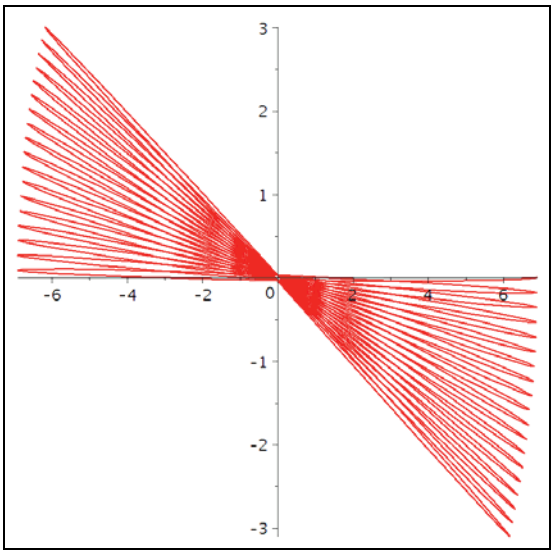

Figure 11. The real-time 300 min run describing the animation plot for the parametric equations of $x(t)$ versus $y(t)$. This resembles the work of J Opera [6] on geometry aspects of FP.

3. Conclusions

This experiment offers an exact evidence of the spin motion of Earth, in turn revealing a technical and precise information that the Earth's surface is not an inertial frame of reference. The calculation worksheet may be filled with just the latitude information for the calculation of all the parameters, plot, and anima-

Figure 12. Exemplar sandplot photo. Courtesy: global.rakuten.com

tion. Students may plan a model/project with suitable/feasible pendulum specifications (especially mass and string length). In a short time, fine and precise calculations, plots, and animations are catered by MAPLE can be boost the effective design of the actual model/project. Miniature table top models of FP are also feasible projects for science exhibitions. Toying with various parameters in the MAPLE animation programme offers a matchless learning experience.

Suggested Reading

[1] The Pendulum of Foucault of the Panthéon, Ceremony of inauguration by M Chaumié, minister of the state education, burnt the wire of balancing, to start the pendulum. 1902. Paris en images.

[2] Joe Kissell, November 8, 2004, Foucault's Pendulum: Low-tech proof of Earth's rotation, Interesting thing of the day, Retrieved March 21, 2012.

[3] Boris Thiolay, (April 28, 2010), Le pendule de Foucault perd la boule (in French), L'Express.

[4] Foucault's pendulum is sent crashing to Earth, Times Higher Education, 13 May 2010. Retrieved March 21, 2012.

[5] V B Braginsky, A G Polnarev and K S Thorne, *Phys. Rev. Lett.*, Vol.53, p.863, 1984.

[6] J Opera, *The American Mathematical Monthly JSTOR*, P515-5222, 1995.