Department Of Statistics

METHODS TO SOLVE NETWORK ANALYSIS

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Topics to be covered:

- Introduction
- Methods for solving network analysis
- Example

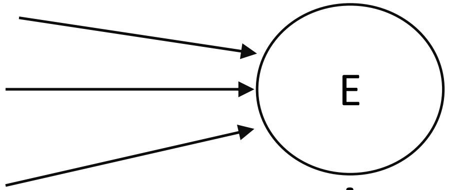
Network:

A network is a graphical representation of arrows and the nodes for showing the logical sequence of various activities to be performed to achieve project objectives. The analysis/study of this network is called as network analysis. Network analysis is used for planning, scheduling and controlling large and complex projects, this technique is based on the representation of the project as a network of activities.

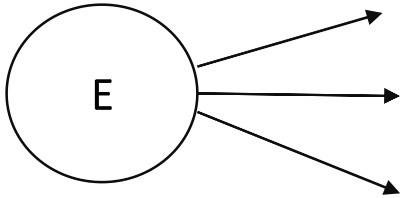
Components of network:

- Event: Events in the network diagram represents the project milestones such as the start or the end completion of an activity. Events are usually represented by circles. There are two type of events→a)Merge event
  b)Burst event

a) <u>Merge event</u>: An event which represents the joint completion of more than one event is known as merge event.



b) <u>Burst event</u>: An event representing the beginning of more than two activity is called as burst event.



- <u>Activity:</u> Activity in the network diagram represents project operations to be conducted. Each activity consumes resources like time, money, raw material, effort, etc. An arrow is commonly used to represent activities.

a) Predecessor activity: An activity which must be completed before one or more activities start is known as predecessor activity.
b) Successor activity: An activity which starts immediately after the completion of one or more of other activity is called as successor activity.
c) Dummy activity: The activity does not consumes either resources or time is called as dummy activity.

❖ There are two methods to solve the **Network Analysis:**

1) Critical path method
2) Program evaluation and review technique

Critical Path Method (CPM):

The objective of critical path analysis is to estimate the total project duration and to assign starting and finishing time to all activities involved in the project. CPM consist of following steps:

1) Calculate the time schedule for each activity: It involves the determination of the time by which an activity must begin and the time before which it must be completed.
2) Calculate the time schedule for the completion of the entire project: It involves the calculation of project completion time.
3) Identify the critical activity and finding critical path: Critical activities are the one which must be started and completed according to the schedule or else the project may be delayed. The path containing these activities is the critical path.

Time calculation in network:

1) Earliest start and finish time: Let zero(0) be the starting point of the project. This will then become the earliest start time(ES) for the first activity. For the given ES for an activity the earliest finish time (EF) of that activity is simply ES+ activity time.

   Thus if ES_i denote the earliest start time of all the activities emitting from node i and t_{ij} is the estimated time of activity (i,j) then,

   ES_i= max(ES_i + t_{ij}) ; for all the defined (i,j) activities

   Note that ES_j=EF_i for the starting and the ending event.

2) Latest start and finish time: Suppose that the we have the target time for the completion of the given project. This time is called as latest finish time(LF) for the final activity. The latest start time(LS) is the least time at which an activity can start if the target is to be maintained. This means that the least start time(LS) for the final activity is LF-activity time.

Thus if LF_j is the latest finish time of all the activities emitting from event i and t_{ij} is the estimated time of the activity(i,j) then,

$$EF_i = \min(LF_j - t_{ij}); \text{ for all the defined (i,j) activities}$$

Forward pass method:

In this method, calculations starts from the initial event and then we proceed through the events in an increasing order of event number and end at the final event. At each event it's earliest occurrence time and earliest start and finish time for each activity that begins at that event is calculated. When calculation ends at final event it's earliest occurrence time gives the earliest possible completion time of the project. The steps for forward pass method are as follows:

Step 1: Set the earliest occurrence time of initial event to 0 i.e., E_1=0 for i=1

Step 2: Calculate the earliest start time for each activity that begins at event i. This is equal to the earliest occurrence time of event i (tail event) i.e.,

ES_{ij}=E_i ; for all activities (i,j) starting at event i

Step 3: Calculate the earliest finish time of each activity that begins at event i. This is equal to the earliest start time of the activity + the duration of activity i.e.,

EF_{ij}= ES_{ij}+t_{ij} = ES_i+t_{ij} ; for all activities (i,j) starting at event i

Step 4: Proceed to the next event say j (j>1). Calculate the earliest occurrence time for event j. This is the maximum of the earliest finish time of all activities ending into that event i.e.,

E_j=max(EF_{ij})=max($E_i + t_{ij}$) ; for all emitting predecessor activities

Step 5: If j=N i.e., the final event then earliest finish time for the project i.e., the earliest occurrence time E_N for the final event is given by

E_N=max(EF_{ij})=max(E_{N-1}+t_{ij}) ,for all terminating activities

<u>Backward pass method</u>:

In this method calculation starts from the final event N(say) we proceed through the events in the decreasing order of event numbers and end at the initial events(i.e. event 1). At each event latest occurrence time and latest finish and start time for each activity i.e. terminating at that event is calculated. The procedure continues till the initial event. This method may be summarized as follows:

<u>Step 1</u>: Set the latest occurrence time of last event N equal to it's earliest occurrence time(known from forward pass method) i.e., L_N=E_N ; j=N

<u>Step 2</u>: Calculate the latest finish time of each activity which end at event j. This This is equal to latest occurrence time of final event i.e.,

$$LS_{ij}=L_j \quad ; \text{for all activities(i,j) ending at event j}$$

<u>Step 3</u>: Calculate the latest start time for all activities ending at j. This is obtained by subtracting the duration of the activities from the latest finish time of the activity.

LS_{ij}=LF_{ij}-t_{ij}=L_j-t_{ij} ,for all activities(i,j)ending at event j

Step 4: Proceed backward to the event in the sequence that increases j by 1 say i. Calculate the latest occurrence time of event i(i<j). This is the minimum of the latest start time of all activities from the event

L_i=min(LS_{ij})=min(L_j-t_{ij}) ,for all immediate successor activities

Step 5: If j=1 i.e., the initial event then the last finish time of the project i.e., latest occurrence time L_1 for the initial event is given by,

L_1=min(LS_{ij})=min(L_{j-1}-t_{ij}) ,for all immediate successor activities

Float (Slack):

Float of an activity is the amount of activity time that can be increased or delayed without delaying project completion time. The float of an event is the difference between its latest occurrence time L_i and the earliest occurrence time E_i i.e.,

Event float=L_i-E_i

There are three type of float:

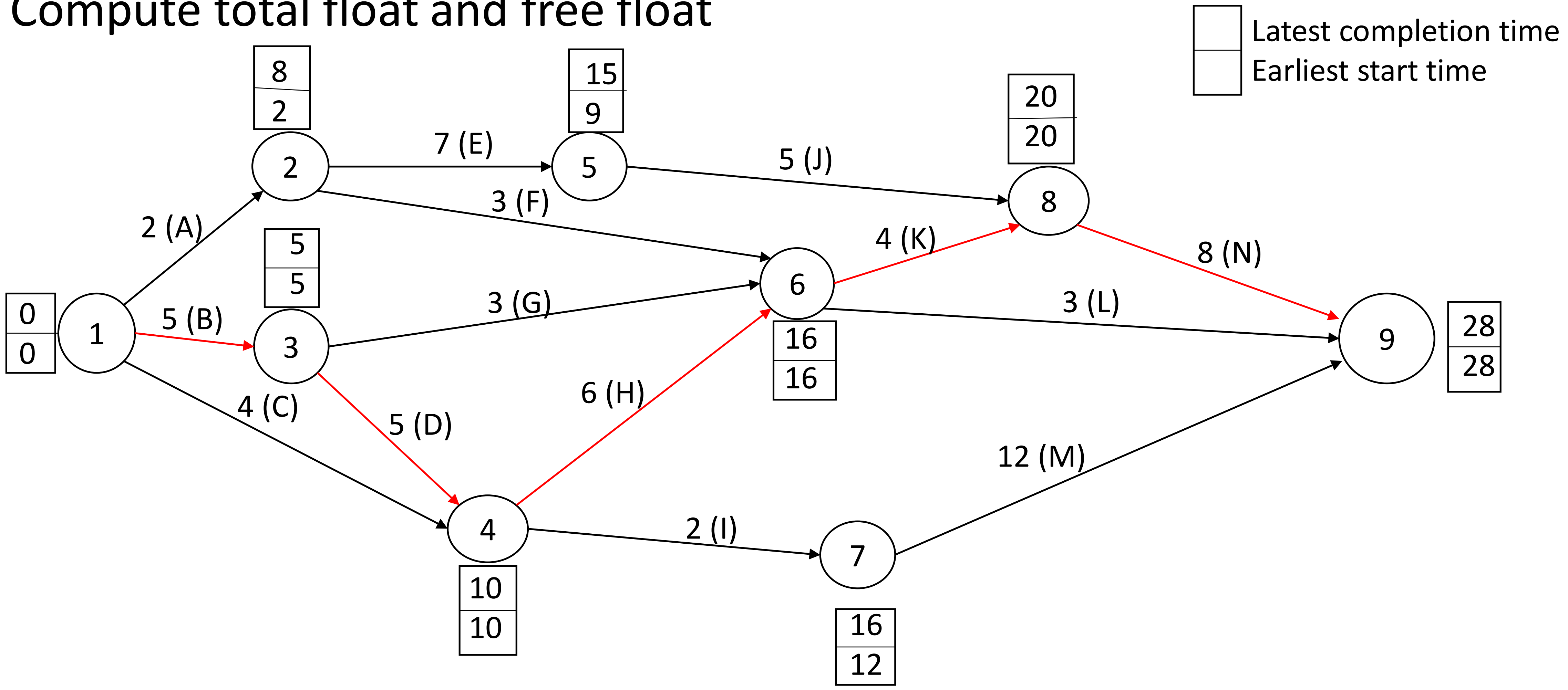
i. Total float: The total float of an activity represents the amount of time by which an activity can be delayed in the project completion time. In other words, it refers to the amount of the free time associated with an activity which can be used before, during or after the performance of activity.
ii. Free float: Free float is that fraction from total float of an activity which can be used for rescheduling the activity without affecting the succeeding activity.
iii. Independent float: Independent float is defined as the amount of time by the start of an activity which can be delayed without affecting the earliest start time of any immediately successor activity assuming that the preceding activity has finished at latest finish time.

Example: Let us consider the work of the project as given in the table

Activity	Immediate predecessor activity	**Duration (days)**
A	-	2
B	-	5
C	-	4
D	B	5
E	A	7
F	A	3
G	B	3
H	C,D	6
I	C,D	2
J	E	5
K	F,G,H	4
L	F,G,H	3
M	I	12
N	J,K	8

Here we will finding:

1. Construction of network diagram
2. Determine critical path and project completion time
3. Compute total float and free float



Here critical activities are B-D-H-K-N

Critical path = 1-3-4-6-8-9

Total path duration = 5+5+6+4+8 = 28 days

Activity	Duration (t_{ij})	ES_i	ES_j	LC_i	LC_j	TF_{ij}=LC_j-ES_i-t_{ij}	FF_{ij}=ES_j-ES_i-t_{ij}
A	2	0	2	0	8	6	0
B	5	0	5	0	5	0	0
C	4	0	10	0	10	6	6
D	5	5	10	5	10	0	0
E	7	2	9	8	15	6	0
F	3	2	16	8	16	11	11
G	3	5	16	5	16	8	8
H	6	10	16	10	16	0	0
I	2	10	12	10	16	4	0
J	5	9	20	15	20	6	6
K	4	16	20	16	20	0	0
L	3	16	28	16	28	9	9
M	12	12	28	16	28	4	4
N	8	20	28	20	28	0	0

Program evaluation review technique (PERT):

PERT is a probabilistic method, where the activity times are represented by a probability distribution. This distribution of activity time is based on 3 different time estimates made for each activity, which are as follows:

i. Optimistic time(t_0): The shortest possible time(duration) in which any activity can be performed or completed assuming under favorable condition.
ii. Most likely time(t_m): It is the estimation of normal time an activity could take. It is the time that would occur most often to complete an activity. Obviously it is the complete time that would occur most frequently i.e., the modal value.
iii. Pessimistic time(t_p): The longest time required to perform any activity under extremely bad condition or if everything goes wrong

From these 3 times, we can calculate expected time of an activity by using

$$t_e = {}^{t_0 + 4t_m + t_p}/_{6}$$

Variance of an activity can be calculate by

$$\sigma^2 = ({}^{t_p - t_0}/_{6})^2$$

Example:

The following table shows the jobs of a network along with their time estimate

Activity	Estimated duration (weeks)		
	Optimistic time(t_0)	Most likely time(t_m)	Pessimistic time(t_p)
1-2	1	7	13
1-6	2	5	14
2-3	2	14	26
2-4	2	5	8
3-5	7	10	19
4-5	5	5	17
6-7	5	8	29
5-8	3	3	9
7-8	8	17	32

Here we will be finding:

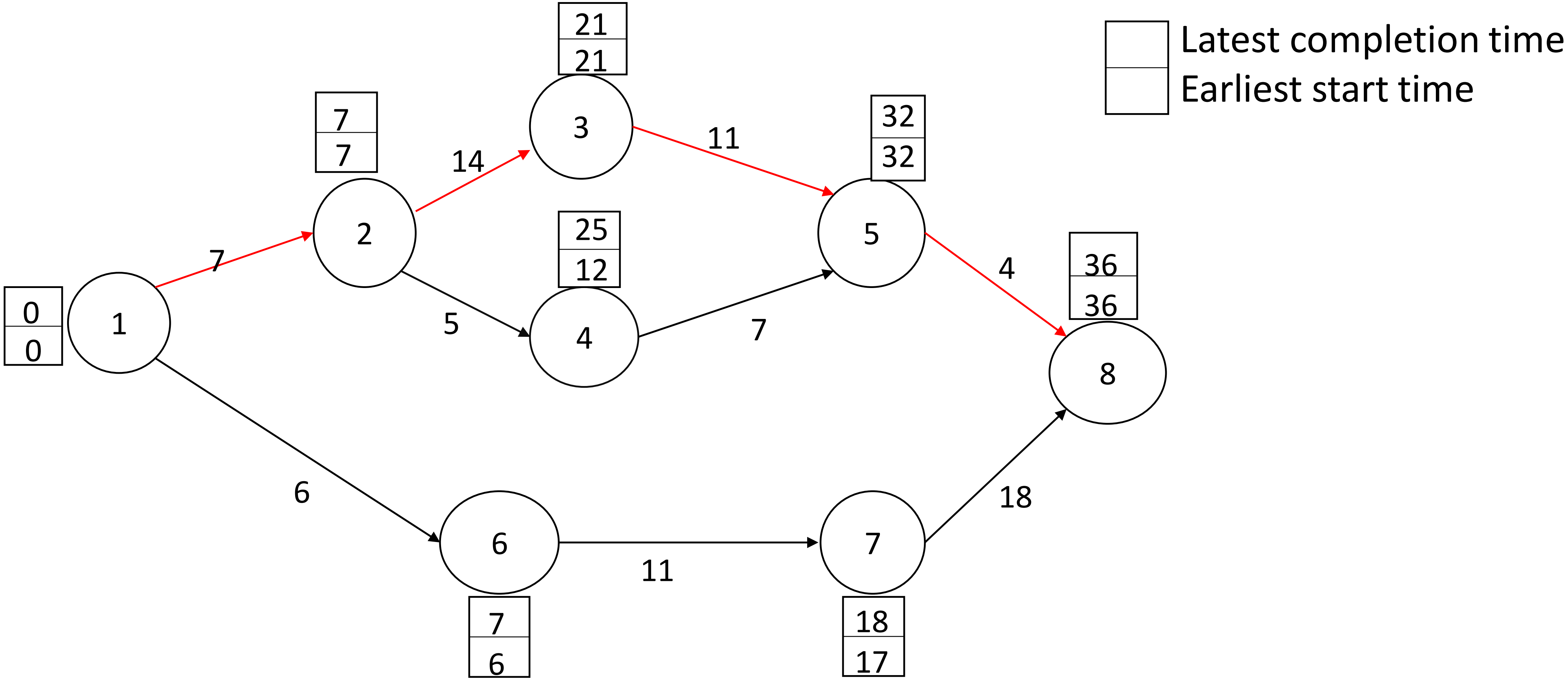
1. Draw network diagram.
2. Find the expected duration and variance of each activity.

3. Calculate the earliest and latest occurrence of each event.

4. Calculate expected project length.

5. Calculate the variance and standard deviation of project length.

6. Find the probability of the project completing in 40 days.



Activity	Estimated duration (weeks)			t_e= $t_0+4t_m+t_p/$	σ^2= $({}^{t_p-t_0}/_6)^2$
	t_0	t_m	t_p		
1-2	1	7	13	7	4
1-6	2	5	14	6	4
2-3	2	14	26	14	16
2-4	2	5	8	5	1
3-5	7	10	19	11	4
4-5	5	5	17	7	4
6-7	5	8	29	11	16
5-8	3	3	9	4	1
7-8	8	17	32	18	16

From the network diagram we can see the critical path is 1-2-3-5-8

Expected project duration = 7+14+11+4 = 36 weeks

Project length variance = σ^2 = 4+16+1+4 = 25

Project length Standard deviation = $\sqrt{\sigma^2}$ = 5

The probability that the project will be completed in 40 days is given by P(Z≤D)

$$D=\frac{T_S-T_e}{\sigma} = \frac{40-36}{5} = 0.8$$

$$P(Z\leq 0.8)=0.7881$$

References:

1. Network analysis in project planning
2. https://en.Wikipedia.org/wiki/Critical_path_method
3. https://acqnotes.com/acnote/task/pert-analysis

Thank You. ☺