NKT/KS/17/5057

Bachelor of Science (B.Sc.) Semester—I (C.B.S.) Examination
STATISTICS (Probability Theory)
Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 50

Note :— ALL questions are compulsory and carry equal marks.

1. (A) Define the term probability according to :

   (i) Classical approach

   (ii) Relative frequency approach

   (iii) Axiomatic approach.

   (B) Prove the following :

   If $A_1, A_2, \ldots, A_n$ are any n events then

   $P(A_1 \cup A_2 \cup \ldots \cup A_n) \leq P(A_1) + P(A_2) + \ldots + P(A_n)$

   $P(A_1 \cap A_2) \ldots \cap A_n) \geq P(A_1) + P(A_2) + \ldots + P(A_n) - (n - 1)$. 5+5

OR

   (E) Define :—

   (i) A random experiment

   (ii) Sample space

   (iii) An event

   (iv) Mutually exclusive events

   (v) Exhaustive events.

   (F) One urn contains three red balls, two white balls and one blue ball. Another urn contains one red ball, two white balls and 3 blue balls. One ball is selected at random from each urn.

   (i) Describe the sample space for this experiment.

   (ii) Find the probability that both balls are blue.

   (iii) Find the probability that both balls are of the same colour. 5+5

2. (A) Define partition of the sample. State and prove Baye's theorem.

(B) B_1, B_2 and B_3 are mutually exclusive events :

$P(B_i) = 1/3, \ i = 1, 2, 3$

$P(A \mid B_i) = i/6, \ \ i = 1, 2, 3$

Find :

(i) $P(A)$

(ii) $P(B_1 \mid A)$. 5+5

OR

(E) Define conditional probability. State and prove multiplicative law for n events.

(F) Two fair dice are thrown. Given that the two faces do not show the same number, find :—

(i) the probability that the sum of the faces is even.

(ii) the probability that one die shows face number 1. 5+5

3. (A) Define mathematical expectation of :

(i) a random variable

(ii) a function of random variable.

State and prove properties of mathematical expectation.

(B) Let X be a r.v. with p.d.f.

$f(x) = 2(1 - x) \quad 0 < x < 1$

$\quad = 0$ elsewhere

Find :

(i) $E(X)$

(ii) $V(X)$

(iii) $P[0 < x < 0.5]$ 5+5

OR

(E) Define c.d.f. of a random variable X. State and prove its properties.

(F) Two coins are tossed. Let X denote the number of heads obtained :

(i) Derive the p.m.f. of X

(ii) Find the c.d.f. and draw its graph. 5+5

4. (A) Define the moment generating function of a random variable X. Explain how moments can be obtained from the m.g.f. Discuss the effect of change of origin and scale on the m.g.f. If m.g.f. of a random variable X is given by $M_X(t) = e^{4t+8t^2}$, find the mean and variance of X. 10

OR

(E) Define probability generating function of a non-negative, integer valued random variable X. Explain how mean and variance can be obtained from it.

(F) Define :—

(i) Mean

(ii) Mean deviation from mean

(iii) Standard deviation

(iv) β_1

(v) β_2 5+5

5. Solve any **TEN** of the following :—

(A) Using axiomatic definition of probability show that $P(\phi) = 0$.

(B) $P(A) = \frac{2}{3}$, $P(\overline{B}) = \frac{1}{3}$. Can A & B be mutually exclusive event ? Justify your answer.

(C) Three pilots Gagan, Ambar and Akash are allotted the same three aircrafts randomly each day. What is the probability that only Gagan gets the same aircraft on two consecutive days.

(D) State True or False :

Given that A and B are independent :

(i) A and $\overline{B}$ are independent

(ii) $\overline{A}$ and $\overline{B}$ are independent.

(E) State True or False :

Given A and B are independent :

(i) A and B are mutually exclusive

(ii) $P(A|B) = P(A)$

(F) State True or False :

(i) Mutual independence of n events $\Rightarrow$ pairwise independence of n events.

(ii) Pairwise independence of n events $\Rightarrow$ mutual independence of n events.

(G) Let $f(x) = Cx(1 - x)$, $0 < x < 1$

$= 0$ elsewhere

be the p.d.f. of a r.v.X. Determine C.

(H) Let $f(x) = x/18$, $0 < x < 6$

$= 0$ elsewhere

be the p.d.f. of a r.v. X. Find $P[0 < X < 3]$.

(I) For a r.v. X, $E(X) = 4$, $V(X) = 2$

Find $E(X^2 - 3X)$

(J) Define mode of a r.v. X.

(K) If $M_x(t) = \left(\frac{2+e^t}{3}\right)^6$, find E(X).

(L) If the p.g.f. of a r.v. X is given by :

$$P_X(s) = \frac{s}{2-s} \quad |s/2| < 1$$

then find E(X). 1×10=10