B.A./B.Sc. (Statistics) Semester—I (C.B.S.) Examination

STATISTICS

(Probability Theory)

Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 50

Note :— All questions are compulsory and carry equal marks.

1. (A) Define with an example :
   (i) A random experiment
   (ii) Sample space
   (iii) An event
   (iv) Exhaustive events
   (v) Impossible event.

   (B) Give axiomatic and classical definitions of probability. Show that the classical definition of probability satisfies all the axioms of probability. 5+5

OR

   (E) Give relative frequency approach to probability. State its limitations. Let A, B and C be three events in the sample space. State the expression for the events noted below in the context of A, B and C.
   (i) Only A occurs
   (ii) Both A and B but not C occur
   (iii) All three events occur
   (iv) At least one of the three events occur
   (v) At least two events occurs
   (vi) None occurs.

   (F) Two fair dice are thrown. Let A be the event that the sum of the points on the faces shown is odd and B is the event of at least one ace (i.e. number 1). State :
   (i) Complete sample space
   (ii) Events A, B, $\overline{B}$, $A \cap B$, $A \cup B$ and $A \cap \overline{B}$ and find their probabilities.
   (iii) Also find $P(A|B)$ and $P(B|A)$. 5+5

2. (A) State and prove multiplicative law of probability for 3 events.

   (B) Define conditional probability. Show that it satisfies axioms of probability.

   (C) Define mutual independence of n events. Show that the number of conditions to be satisfied for mutual independence of n events is $2^n - n - 1$.

   (D) A husband and wife appear in an interview for two vacancies. The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability that :
   (i) Only one of them will be selected
   (ii) Both of them will be selected
   (iii) None of them will be selected ?
   Assume that the selections of husband and wife are independent. 2½×4=10

OR

   (E) If the events A, B and C are mutually independent, then show that $(A \cup B)$ and C are independent. If $(\overline{A} \cap B) = 0.1$, $P(A \cap \overline{B}) = 0.6$ and $P(A \cap B) = 0.2$, find $P(A|B)$.

   (F) State and prove Bayes' theorem. 5+5

3. (A) Define c.d.f. of a r.v. Show that it is non-decreasing.

The probability mass function of a r.v. X is given below :

X	0	1	2	3	4	5	6	7	8
P[X = x]	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Determine the value of a

(ii) Find $P(X < 2)$, $P(X \geq 6)$, $P(3 < X < 5)$

(iii) Find distribution function of X.

(B) For a discrete r.v. define :

(i) Probability mass function

(ii) Expected value.

Let p.m.f. of a r.v. be given by :

X	–3	6	9
p(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find :

(i) E(X)

(ii) E[2X + 5]

(iii) V(X). 5+5

OR

(E) Find the probability distribution of the r.v. X and also find the following probabilities :

(i) $P[X \geq 4]$

(ii) $P[X \leq 10]$

(iii) $P[X > 10]$

(iv) $P[1 \leq X < 6]$, if the C.D.F of r.v. X is given by :

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{3} & 1 \leq x < 4 \\ \frac{1}{2} & 4 \leq x < 6 \\ \frac{5}{6} & 6 \leq x < 10 \\ 1 & x \geq 10 \end{cases}$$

(F) Let f(x) be the p.d.f. of r.v. X

$$f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that :

$$E[X^r] = \frac{2}{(r+1)(r+2)}$$

Hence find :

(i) Mean

(ii) Variance of r.v. X. 5+5

4. (A) Define

(i) r^{th} raw moment

(ii) r^{th} central moment of a r.v.

Derive the expression for r^{th} central moment in terms of raw moments. Hence obtain the expressions for second, third and fourth central moments in terms of raw moments. 10

OR

(E) Define the m.g.f. of r.v. X. Show that it generates moments about origin.

(F) Define the p.g.f. of a r.v. Show how the mean and the variance of a r.v. can be calculated from it.

(G) The first four moments of a probability distribution about the origin are 1, 4, 10 and 46 respectively. Obtain μ_2, μ_3, β and γ_1 on the basis of information given. Comment upon the nature of the distribution.

(H) Let X be a r.v. with p.g.f. p(s), find the generating function of :

(i) X + 1

(ii) 2X. 2½×4=10

5. Solve any **TEN** of the following :

(A) State the limitations of classical definition of probability.

(B) Define mutually exclusive events.

(C) A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue ?

(D) For any 3 events A, B and C :

$$P\left[(A \cap \overline{B})|C\right] + P\left[(A \cap B)|C\right] = P(A|C)$$

(E) Define partition of the sample space.

(F) Two events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$. Are A and B independent events ?

(G) For what values of a and b will V(aX + b) = V(X) + b ?

(H) Find the constant C such that the function

$$f(x) = \begin{cases} Cx^2 ; & 0 < x < 3 \\ 0 ; & \text{otherwise} \end{cases}$$

is a p.d.f. of X.

(I) If $f(x) = \frac{x}{2}$; $0 < x \le 2$

$= 0$; otherwise

Find $P[1 \le X \le 2]$.

(J) Let X be a r.v. with following p.m.f :

X	0	1	2
p(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Find mode of the distribution.

(K) Define Bowley's coefficient of skewness.

(L) If $f(x) = \frac{1}{2}$; $-1 < x < 1$

$= 0$; otherwise

Find median of the distribution. 1×10=10