Bachelor of Science (B.Sc.) Semester—I Examination

STATISTICS (Probability Theory)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

Note :— All questions are compulsory and carry equal marks.

1. (A) Define with an example :

   (i) A random experiment

   (ii) Discrete and continuous sample space

   (iii) Impossible event

   (iv) Exhaustive events.

   (B) Give axiomatic definition of probability and prove the following :

   (i) $P(\phi) = 0$

   (ii) $P(\overline{A}) = 1 - P(A)$

   (iii) If $A \subset B$ then $P(A) \leq P(B)$. 5+5

OR

   (E) Give classical definition of probability. State its limitations. Also describe relative frequency approach of probability. Let A, B and C be three events in a sample space. State the expressions for the events noted below in the context of A, B and C :

   (i) None occurs

   (ii) Only A occurs

   (iii) At least one of the three events occur

   (iv) All three events occur

   (v) Exactly two occur. 10

2. (A) State and prove Bayes theorem.

   (B) Define conditional probability and show that it satisfies the axioms of probability. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(A \cup B) = 0.9$. Are A and B independent events ? Why ? 5+5

OR

   (E) Define mutually independent events. Suppose that A, B and C are mutually independent events and that $P(A) = 0.5$, $P(B) = 0.8$ and $P(C) = 0.9$. Find the probabilities that :

   (i) All three events occur.

   (ii) Exactly two of the three events occur and

   (iii) None of the events occur.

   (F) State and prove multiplicative law of probability for n events. A box of fuses contains 20 fuses, of which 5 are defective. If 3 of the fuses are selected at random and removed from the box in succession without replacement then find the probability that all the three fuses are defective. 5+5

3. (A) Define cumulative distribution function of a r.v. State and prove its 4 properties. If the r.v. X has the following p.d.f. :

$$f_X(x) = \begin{cases} Cx & 0 \le X \le 2 \\ 0 & \text{otherwsie} \end{cases}$$

use the p.d.f. to find :

(a) The constant C

(b) $P[0 \le X \le 1]$

(c) $P\left[-\frac{1}{2} \le X \le \frac{1}{2}\right]$

(d) The C.D.F. of X. 10

OR

(E) Define expectation of a random variable. If $X_1, X_2, \ldots X_n$ are n random variables and $a_1, a_2, \ldots a_n$ are n constants then show that :

$$E(a_1X_1 + a_2X_2 + \ldots + a_nX_n] = a_1E(X_1) + a_2E(X_2) + \ldots + a_nE(X_n)$$

If the pmf of a r.v. X is :

X	1	3	5
P(x)	2k	3k	4k

where k is constant. Find :

(i) k

(ii) $P(X > 2)$

(iii) E(X)

(iv) C.d.f. of X.

(v) Draw the graphs of pmf and c.d.f. of X. 10

4. (A) Define p.g.f. of a r.v. X. If X is a r.v. which assumes only integral values with probability distribution :

$$P[X = k] = p_k \qquad k = 0, 1, 2 \ldots..$$

$$P[X > k] = q_k \qquad k = 0, 1, 2 \ldots..$$

so that $q_k = p_{k+1} + p_{k+2} + \ldots\ldots$

and two generating functions are :

$$P(s) = p_0 + p_1s + p_2s^2 + \ldots.$$

$$Q(s) = q_0 + q_1s + q_2s^2 + \ldots.$$

then for $-1 < s < 1$, prove that :

$$Q(s) = \frac{1 - P(s)}{1 - s}.$$

(B) Define Kurtosis. Discuss various types of kurtosis with the help of figures. Also give the formula to measure kurtosis. If the coefficient of kurtosis and the fourth central moment of a probability distribution are respectively 3 and 27 then find its standard deviation. 5+5

OR

(E) Define :

(i) r^{th} raw moment

(ii) r^{th} central moment of a r.v.

Derive an expression for r^{th} central moment in terms of raw moments. Hence obtain the expressions for second, third and fourth central moments in terms of raw moments. 10

5. Solve any ten of the following :

(A) Define mutually exclusive events.

(B) If P(A) = 0.5, P(B) = 0.3. Is it possible to have $P(A \cap B) = 0.6$?

(C) Two cards are drawn randomly out of 52 cards. Find the probability that both are kings.

(D) Define pairwise independent events.

(E) If A and B are independent events. Show that A and $\overline{B}$ are also independent.

(F) Let P(A) = 0.3, P(B) = 0.6. Find P(A/B), when A and B are mutually exclusive.

(G) Define a random variable.

(H) In a class of 100 students, 80 students passed in all subjects, 10 failed in one subject, 7 failed in 2 subjects and 3 failed in 3 subjects. Find the probability distribution of the variable which denotes the number of subjects the student has failed in.

(I) Give an example to show that :

$$E(X^2) \neq [E(X)]^2.$$

(J) State the relationship between mgf and pgf.

(K) Define skewness of the probability distribution of a r.v.

(L) If $M_X(t) = (1 - 2t)^{-10}$ is the mgf of a r.v. then find E(X). 1×10=10