

Bachelor of Science (B.Sc.) Semester-II (C.B.S.) Examination

STATISTICS (PROBABILITY DISTRIBUTIONS)

Compulsory Paper—1

Time : Three Hours]

[Maximum Marks : 50

N.B. :— All the **FIVE** questions are compulsory and carry equal marks.

1. (A) State the p.m.f. of Binomial distribution. Find its first three moments about origin. Hence find μ_2 , μ_3 and β_1 . Comment on skewness of the distribution. 10

OR

- (E) State p.m.f. of Poisson distribution. Find its m.g.f. State and prove additive property of Poisson distribution.

- (F) Find mode of Binomial distribution. Comment on the following statement :

“X follows B(n, 0.5). The distribution of X is Unimodal or Bimodal according as n is even or odd.” 5+5

2. (A) Derive p.m.f. of Hypergeometric distribution. Find mean and variance of this distribution. An Indian Revenue Service (IRS) inspector is to select 3 corporations from a list of 15 for tax audit purposes; of the 15 corporations, 6 earned profits and 9 incurred losses during the year. Find the probability that out of the number of corporations in the selected three, two incurred losses during the year. 10

OR

- (E) State p.m.f. of Negative Binomial distribution. Find its mean and variance. In an experiment of throwing a fair die if the face numbered 4 turns up it will be treated as a success otherwise a failure. What is the probability of getting 4th success in the 6th trial ? 10

3. (A) State p.d.f. of normal distribution. Derive recurrence relation for even ordered central moments. Hence find μ_2 and μ_4 . Comment on Kurtosis of a distribution. Also show that the distribution is symmetric about its mean by calculating its mean and mode. 10

OR

- (E) Define standard normal variable. Write its p.d.f. Find m.g.f. of the distribution. If Z is a standard normal variable, show that :

$$F(-z) = 1 - F(z) \text{ in usual notation.}$$

- (F) State the chief characteristics of normal distribution. If X_1 and X_2 are independent normal variables with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2) respectively. Show that $X_1 + X_2$ is a normal variable. Write parameters of the distribution of $X_1 + X_2$. 5+5

4. (A) State p.d.f. of exponential distribution and find its mean and variance.

- (B) State and prove lack of memory property of exponential distribution.

- (C) State p.d.f. of Beta distribution of second kind. Find rth moment of this distribution.

- (D) State and prove additive property of Gamma distribution with one parameter.

2½×4=10

OR

- (E) State p.d.f. of Gamma distribution with two parameters. Find its m.g.f. and hence find mean and variance of the distribution.
- (F) State p.d.f. of Beta distribution of first kind. Find its r^{th} raw moment and hence find mean and variance of the distribution. 5+5

5. Solve any **TEN** questions :

- (A) Define Bernoulli trial.
- (B) Find m.g.f. of Bernoulli distribution.
- (C) If m.g.f. of the random variable X is $\left(\frac{1+2e^t}{3}\right)^9$. State the probability distribution of X and state its mean.
- (D) Name the discrete distribution which lacks memory and state its p.m.f.
- (E) If the r.v. X follows geometric distribution with parameter $\frac{1}{3}$. Find its mean.
- (F) State p.g.f. of geometric distribution.
- (G) If r.v. X has uniform distribution in the interval $(-2, 3)$ write its p.d.f.
- (H) If r.v. $X \sim N(\mu, \sigma^2)$, show that :

$$P(a \leq X \leq b) = F\left(\frac{b-\mu}{\sigma}\right) - F\left(\frac{a-\mu}{\sigma}\right)$$

where a and b are any constants.

- (I) If random variable X follows uniform distribution with parameters (a, b), $a < b$ then find cumulative distribution function of X.
- (J) State the p.d.f. of Beta distribution of first kind.
- (K) If the r.v. X follows the following probability law :

$$f(x) = \frac{12x^2}{(1+x)^5}, x > 0$$

Identify the distribution of X.

- (L) If X_1 and X_2 are two independent Gamma variables with parameters 4 and 5 respectively. State the distribution of $X_1 + X_2$. 1×10=10