NRT/KS/19/2061

Bachelor of Science (B.Sc.) Semester–II Examination

STATISTICS (PROBABILITY DISTRIBUTIONS)

Optional Paper–1

Time : Three Hours] [Maximum Marks : 50

N.B. :—All **FIVE** questions are compulsory and carry equal marks.

1. (A) Derive the p.m.f of a Binomial distribution. Find its first three raw moments about origin. Hence find μ_2, μ_3 and β_1. Comment on the skewness of Binomial distribution. 10

OR

(E) The average number of customers, who appear at a counter of a certain bank per minute is two. Find the probability that during a given minute :

(i) No customer appears.

(ii) Three or more customers appear.

(F) State and prove additivity property of Poisson distribution.

(G) State p.m.f of a Poisson distribution, find its mean and variance.

(H) State the p.m.f of a Bernoulli distribution. Obtain its m.g.f. Hence find mean and variance of this distribution. 2½×4=10

2. (A) Derive the p.m.f. of a Geometric distribution. State and prove the lack of memory property possessed by this distribution.

An amateur sharp shooter makes several attempts until she hits the target :

(a) If p = 0.4 is the probability of hitting the target. What is the probability of having the first hit at the 3rd attempt ?

(b) What is the probability that the third hit occurs on the 6th attempt ? 10

OR

(E) State p.m.f. of negative Binomial distribution. Find its m.g.f and hence find mean of the distribution.

(F) Derive m.g.f. of Geometric distribution. Hence find its mean and variance.

(G) Derive p.m.f. of Hypergeometric distribution and find its mean.

(H) As a part of air of pollution survey an inspector decides to examine exhaust of six trucks out of a company's 24 trucks. If four of the company's trucks emit excessive amount of pollutants, what is the probability that none of them will be included in the inspector's sample ? 2½×4=10

3. (A) Show that odd ordered central moments of normal distribution are zero. Derive the formula for the recurrence relation for even ordered central moments of this distribution. Hence find μ_2, μ_4 and β_2. Comment on Kurtosis of the distribution. 10

OR

(E) State probability density function of Normal distribution with mean μ and variance σ^2. Define a standard Normal variable. Obtain its probability density function.

(F) Find mode of a Normal distribution.

(G) If X denotes the number that appeared on the face turned up in a single throw of an unbiased die write p.m.f of X and hence find its mean and variance.

(H) Buses arrive at a specified stop at 15 minute intervals starting at 7 A.M. That is they arrive at 7, 7.15, 7.30 and so on.

If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7.30 ? Find the probability that he waits :

(a) Less than 5 minutes for a bus.

(b) At least 12 minutes for a bus. 2½×4=10

4\. (A) State p.d.f of Gamma distribution with parameter λ. Find its m.g.f. and hence find mean and variance of the distribution. State and prove additivity property of this distribution. Also find mode of this distribution. 10

OR

(E) Derive the m.g.f. of exponential distribution. Hence find its mean and variance.

(F) Obtain the r^{th} raw moment about origin for β distribution of first kind. Hence find its mean.

(G) Find m.g.f. of Gamma distribution with two parameters and hence find its mean.

(H) The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 1$:

(i) What is the probability that a repair time exceeds 2 hours ?

(ii) What is the probability that a repair takes time between two to three hours ?

2½×4=10

5\. Solve any **TEN** questions :

(A) Suppose that the average number of accidents occurring weekly on a particular highway equals 3. Calculate the probability that there is at least one accident this week.

(B) A random variable has the m.g.f. $e^{7}(e^{t} - 1)$. Identify its probability distribution. State its p.m.f.

(C) Find mode of the Binomial distribution with $p = \frac{1}{2}$ and $n = 7$.

(D) Give one real life example where geometric distribution is applicable.

(E) State the parameters of Hyper geometric distribution.

(F) Why is the Geometric distribution named so ?

(G) State the area property of a Normal distribution.

(H) If X is a normal variate with mean 1 and variance 4 and Y is another normal variate independent of X, with mean 2 and variance 3. What is the distribution of X + Y ? State mean and variance of distribution of X + Y.

(I) The distribution of a variable X is given by the law :

$f(x) = Ke^{-\frac{1}{2}\left(\frac{x-100}{5}\right)^2}$, where $-\infty < x < \infty$. Write the value of :

(i) constant K

(ii) the mean

(iii) the median

(iv) the mode

(v) Standard deviation.

(J) State the p.d.f of beta distribution of second type.

(K) State the continuous distribution for which mean is equal to variance.

(L) Comment on the skewness of Gamma distribution with one parameter. 1×10=10