Bachelor of Science (B.Sc.) Semester—II Examination

STATISTICS (Probability Distributions)

Optional Paper—1

Time : Three Hours] [Maximum Marks : 50

N.B. :— All questions are compulsory and carry equal marks.

1. (A) Derive p.m.f. of Binomial distribution. In usual notation, show that for Binomial distribution :

$$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]$$

Hence find first four central moments. Also find β_1 and β_2 and comment on skewness and kurtosis of Binomial distribution. 10

OR

(E) State p.m.f. of Poisson distribution and find its mean and variance. Also find μ_3 and comment on skewness of the distribution.

(F) Derive the recurrence relation for probabilities of Poisson distribution. Let X be a Poisson variable with P[X = 1] = 0.4 and P[X = 2] = 0.2. Evaluate P[X = 0] and P[X = 3]. Also find mean and variance of X. 5+5

2. (A) State p.m.f. of negative Binomial distribution. Find its m.g.f. and hence find mean of the distribution.

(B) State and prove lack of memory property of Geometric distribution.

(C) Derive p.m.f. of Hypergeometric distribution and find its mean.

(D) Among the 16 applicants for a job, 10 have college degrees. If three of the applicants are randomly chosen for interviews, what are the probabilities that :

(i) None has a college degree

(ii) All three have college degrees ? 2½×4=10

OR

(E) Derive p.m.f. of Geometric distribution. Find its m.g.f. and hence find mean and variance of this distribution. State the relationship between geometric distribution with parameter p and negative Binomial distribution with parameters r and p. Using mean and variance of Geometric distribution state mean and variance of negative binomial distribution. 10

3. (A) State the p.d.f. of normal distribution. Show that all odd ordered central moments of normal distribution vanish.

(B) State p.m.f. of continuous uniform distribution. Find its c.d.f. Also find mean of the distribution.

(C) Show that a linear combination of independent normal variables is a normal variable.

(D) Find mode of normal distribution. 2½×4=10

OR

(E) Define Standard Normal Variable. State its p.d.f. Find m.g.f. of standard normal distribution and hence find its mean and variance.

(F) Explain Area Property of Normal distribution. 5+5

4. (A) State p.d.f. of Gamma distribution with parameter λ. Find its m.g.f. and hence find mean and variance of the distribution. Also find mode of Gamma distribution. State and prove additive property of Gamma distribution. 10

OR

(E) State and prove lack of memory property of exponential distribution.

(F) Derive m.g.f. of exponential distribution and find its mean and variance.

(G) Obtain r^{th} raw moment about origin for Beta distribution of 2^{nd} kind and hence find its mean.

(H) Find m.g.f. of Gamma distribution with two parameters and hence find its mean. 2½×4=10

5. Solve and **TEN** questions :

(A) If X follows exponential distribution with p.d.f. :

$$f(x) = \begin{cases} \theta e^{-\theta x}, & \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

If its mean is $\frac{1}{3}$, then state its variance.

(B) State the continuous probability distribution for which mean and variance are equal.

(C) Comment on skewness of Gamma distribution with one parameter.

(D) If $M_X(t) = e^{2(e^t - 1)}$. State the probability distribution of X with value of the parameter.

(E) Find p.g.f. of Bernoulli distribution.

(F) Define a Bernoulli trial.

(G) Write p.m.f. of discrete uniform distribution.

(H) If $f(x) = \frac{1}{N}$, $x = 1, 2, ...N$ find mean.

(I) If X has m.g.f. $M_X(t) = e^{4t + \frac{9t^2}{2}}$. State probability distribution of X with parameters.

(J) Find mean of hypergeometric distribution with n = 3, N = 16, k = 10.

(K) For sampling without replacement from finite population state the probability distribution of the number of successes in n trials.

(L) Why is the name negative binomial distribution ? 1×10=10