B.A./B.Sc. (Statistics) Semester—III (C.B.S) Examination

STATISTICS

(Statistical Methods)

Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 50

Note :— All questions are compulsory and carry equal marks.

1\. (A) Define joint p.m.f. of two random variables X and Y. Also define their marginal p.m.f.s. When are these two random variables said to be independent ? Let the joint p.m.f. of X and Y be defined by

$$f(x, y) = \frac{x+y}{21}, \; x = 1, 2, 3; \; y = 1, 2$$

Find marginal p.m.f.s of X and Y. Also find E(X) and V(X). Check whether the random variables X and Y are independent. 10

OR

(E) Let X_1 and X_2 have the joint p.d.f.

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find :

(i) The marginal densities of X_1 and X_2.

(ii) The conditional p.d.f. of X_1 given $X_2 = x_2$.

(iii) The conditional mean and conditional variance of X_1 given $X_2 = x_2$.

(iv) $P\left(0 < X_1 < \frac{1}{2} / X_2 = \frac{3}{4}\right)$

(v) $P\left(0 < X_1 < \frac{1}{2}\right)$. 10

2\. (A) State the p.d.f. of Bivariate normal distribution of (X, Y). Find its m.g.f. and hence find E(X) and E(Y). Show that the correlation coefficient between (X, Y) is ρ. 10

OR

(E) Derive the probability function of trinomial distribution. Also find its m.g.f.

(F) State the joint p.d.f. of Bivariate normal distribution. Find the marginal density function of X. 5+5

3\. (A) Let X_1 and X_2 be independent r.vs following normal distribution with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2) respectively. Find the probability distribution of $a_1X_1 + a_2X_2$.

(B) If X follows Poisson distribution with parameter 4. Find the p.m.f. of $Y = \sqrt{X}$.

(C) If X is a standard normal variable, find the p.d.f. of $Y = X^{1/3}$.

(D) If X_1 and X_2 are independent random variables having exponential distributions with the same parameters θ, find the probability density of the random variable $X_1 + X_2$. 2½×4=10

OR

(E) Let X_1 and X_2 be two independent random variables having identical gamma distributions with parameters (α, β).

(i) Find the joint probability density of the random variables $Y_1 = \frac{X_1}{X_1 + X_2}$ and $Y_2 = X_1 + X_2$.

(ii) Find and identify the marginal density of Y_1.

(F) If the joint probability distribution of X_1 and X_2 is given by :

$$p(x_1, x_2) = \frac{x_1 x_2}{36}, \quad \begin{matrix} x_1 = 1, 2, 3 \\ x_2 = 1, 2, 3 \end{matrix}$$

Find :

(i) The joint distribution of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

(ii) The marginal probability distribution of Y_1. 5+5

4. (A) Define F-statistic. Derive its p.d.f. Find r^{th} raw moment about origin of F-distribution and hence find its mean and variance. 10

OR

(E) Define Fisher's t State p.d.f. of t-distribution. Derive p.d.f. of t^2.

(F) Define Chi square statistic. State its p.d.f. and find mode of Chi square distribution. 5+5

5. Solve any **TEN** from the following :

(A) Define bivariate m.g.f.

(B) Define correlation coefficient and state its limits.

(C) Show that cov (aX, bY) = ab cov (X, Y).

(D) If (X_i, Y_i) are bivariate normal with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ for i = 1, 2, ...n. State the probability distribution of $(\overline{X}, \overline{Y})$.

(E) If r.v. (X, Y) follows bivariate normal distribution with parameters $(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, state the conditional p.d.f. of Y given X = x.

(F) State the p.d.f. of multinomial distribution.

(G) Define a "random sample".

(H) Define a 'statistic'.

(I) Define sampling distribution.

(J) State m.g.f. of Chi square distribution.

(K) If $X \sim F(n_1, n_2)$, state the probability distribution of $\frac{1}{X}$.

(L) Comment : F-distribution is highly positively skewed distribution. 1×10=10