NRT/KS/19/2091

Bachelor of Science (B.Sc.) Semester—III Examination

STATISTICS (STATISTICAL METHODS)

Optional Paper—I

Time : Three Hours] [Maximum Marks : 50

N.B. :— ALL questions are compulsory and carry equal marks.

1. (A) The joint pmf of X and Y is :

$$p(x, y) = \frac{xy^2}{K} \quad x = 1, 2, 3; y = 1, 2$$
$$= 0 \quad \text{elsewhere}$$

(i) Determine the constant K.

(ii) Find the marginal pmf of X and Y.

(iii) Are X and Y stochastically independent ?

(iv) If p(x, y) is not defined for (x = 3, y = 2), what will be the new value of K ? 10

OR

(E) X and Y are continuous random variables having joint pdf f(x, y) = 8 xy $0 \le x \le y \le 1$

(i) Find the marginal pdf[s] of X and Y.

(ii) Compute μ_X, μ_Y, σ_X^2 σ_Y^2. 10

2. (A) State the pdf of bivariate normal distribution of a pair of random variables X and Y. Derive mgf of X and Y and hence find mgf of X alone and mgf of Y alone. 10

OR

(E) State the pmf of trinomial distribution. Find its mgf and hence deduce the mgf of marginal distributions of random variables. Also state means and variances of these distributions. Three machines A, B and C produce certain article in the ratio 2 : 3 : 4. From a day's production; if 7 articles are picked up at random, what is the probability that 3 are from machine A and 2 from machine B ? 10

3. (A) Define a random sample and state the steps involved in drawing random sample of size n from exponential distribution with parameter θ = 4.

(B) If X_1, X_2, ... X_n is a random sample from exponential distribution with parameter θ, determine the probability distribution of $Y = \sum_{i=1}^{n} X_i$ using mgf technique. 5+5

OR

(E) Let X and Y be two independent random variables following gamma distribution with respective parameters m and n, then find the distribution of U = X/X+Y.

(F) Let X be a random variable with probability distribution

$$p(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1} \quad x = 1, 2, 3, ...$$

Find the probability distribution of

(i) $Y = \frac{1}{X}$

(ii) $U = X^2$. 5+5

4. (A) Derive an expression for even ordered central moments of t-distribution with n degrees of freedom. Hence find variance and fourth central moment.

(B) Define Student's-t and Fisher's-t. Show that Student's-t is a particular case of Fisher's-t. 5+5

OR

(E) Define chi-square statistic and derive its probability distribution. Show that the sum of independent chi-square variables is also a chi-square variable. 10

5. Solve any ***ten*** questions out of the following :

(A) If μ_{rs} denotes the joint central $(r, s)^{th}$ moment of random variables X and Y then show that :

$$-\sqrt{\mu_{02} \cdot \mu_{20}} < \mu_{11} < \sqrt{\mu_{02} \cdot \mu_{20}} .$$

(B) The joint probability distribution of a pair of random variables (X, Y) is given in the following table :

Y \ X	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Find the marginal pdf^{s} of X and Y. Are X and Y stochastically independent ?

(C) Let f(x, y) = x + y ; 0 < x < 1, 0 < y < 1. Verify whether f(x, y) is a probability density function.

(D) Define a Statistic.

(E) If a r.v X has cdf F(x), then show that F(x) is uniformly distributed on (0, 1).

(F) If $X \sim N(\mu, \sigma^2)$, then state the distribution of $\left(\frac{X-\mu}{\sigma}\right)^2$.

(G) If (X, Y) follow bivariate normal distribution with parameters $\mu_X = 20$, $\mu_Y = 50$, $\sigma_X^2 = 81$, $\sigma_y^2 = 100$ and $\rho = 0.7$. Find V(Y/X = 30) and V(Y/X = 10).

(H) 'X and Y follow trinomial distribution with correlation coefficient 0.6.' Comment on the validity of this statement with reason.

(I) If two correlated random variables X and Y follow bivariate normal distribution with $\mu_X = 20$ and $\mu_Y = 25$, then state the values of Y and X for which $E(X/Y) = \mu_x$ and $E(Y/X) = \mu_Y$.

(J) Why the central moments of t-distribution are same as raw moments about origin ?

(K) If the variance of t-distribution is 1.5, what will be its fourth central moment ?

(L) If X and Y are independent non-negative variables such that X + Y and X follow chi-square distribution with n and m degrees of freedom, then state the distribution of Y . (n > m). 1×10=10