Bachelor of Science (B.Sc.) Semester-III Examination

STATISTICS (STATISTICAL METHODS)

Optional Paper—I

Time : Three Hours] [Maximum Marks : 50

Note :— ALL questions are compulsory and carry equal marks.

1. (A) Two random variables X and Y have their joint probability density function

$$f(x, y) = \begin{cases} K(4 - x - y) & 0 \le x \le 2; 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find :

(i) The constant K

(ii) Marginal pdfs

(iii) Conditional density functions of Y/X and X/Y

(iv) V(X), V(Y) and Cov(X, Y). 10

OR

(E) In an experiment, a regular tetrahedron with faces numbered 1 to 4, is tossed twice. Let X denote the number on the downturned face of the first toss and Y the number on downturned face in second toss.

(i) Write in tabular form the joint pmf of X and Y.

(ii) Find the marginal pmfs of X and Y.

(iii) Also find E(X), E(Y), V(X), V(Y), Cov(X, Y) and ρ_{XY}. 10

2. (A) State the pmf of trinomial distribution of random variables X and Y. Find the conditional distribution of X given Y. Also find the conditional mean and variance of X. 10

OR

(E) State the mgf of (X, Y) ~ B $\vee$ N (μ_1, μ_2, σ_1^2, σ_2^2, ρ). Using the mgf, find E(X), E(Y), V(X), V(Y), Cov(X, Y) and correlation coefficient between X and Y. 10

3. (A) If X_1 and X_2 are independent standard normal variables. Find the pdf of :

(i) $Y = X_1^2 + X_2^2$ and

(ii) $U = X_1^2/X_2^2$ 10

OR

(E) Let X be a continuous random variable having pdf :

$$\begin{aligned} f(x) &= 2x \quad 0 < x < 1 \\ &= 0 \quad \text{elsewhere} \end{aligned}$$

Find the probability distribution of $Y = 8X^3$.

(F) Using mgf technique, show that the sum of 'k' independent Bernoulli variables with constant probability of success 'p' follows binomial distribution with parameters k and p. 5+5

4. (A) Define F-statistic and derive its distribution. Also find its mean. 10

OR

(E) If X and Y are independent chi-square variables with m and n degrees of freedom, then show that :

(i) $\frac{X}{X+Y} \sim \beta_1 (m/2, n/2)$

(ii) $\frac{X}{Y} \sim \beta_2 (m/2, n/2)$ 10

5. Solve any **TEN** questions :

(A) Show that correlation coefficient between two independent random variables is zero. Is the converse true ?

(B) If X ~ b (n, p) and Y ~ (n, 1 – p) then state the distribution of X + (n – Y), given that X and Y are independent.

(C) Define mgf of bivariate probability distribution.

(D) State the criteria for random variables to be stochastically independent.

(E) Let the pmf of random variable X be :

X	:	1	2	3	4	5
p(x)	:	0.2	0.3	0.2	0.2	0.1

Find the pmf of | X – 3 |.

(F) If $X_1, X_2, \ldots\ldots, X_n$ is a random sample with mgf of X_i being $M_X(t)$ for all i, then express

$M \sum_{i=1}^{n} X_i(t)$ in terms of $M_X(t)$.

(G) If two positively correlated random variables X and Y follow B $\vee$ N with $\mu_X = 25$ and $\mu_Y = 30$, for what values of X will E(Y/X) > 30 ? and for what values of Y will E(X/Y) < 25 ?

(H) If X and Y follow trinomial distribution with parameters n = 10, $p_1 = 0.3$ and $p_2 = 0.4$, then for what value of Y will E(X/Y) = E(X) ?

(I) If X and Y follow trinomial distribution with parameters n = 8, $p_1 = 0.2$, $p_2 = 0.3$ find Cov(X, Y).

(J) Show that t-distribution is leptokurtic and tends to be mesokurtic as degrees of freedom increase.

(K) For what degrees of freedom will the standard deviation of chi-square variable be same as its mean ?

(L) 'Mode of f distribution is 1.5', comment on the validity of this statement with reason.

1×10=10