NKT/KS/17/5154

Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination

STATISTICS (Statistical Inference)

Paper—I

Time : Three Hours] [Maximum Marks : 50

N.B. :— All the **FIVE** questions are compulsory and carry equal marks.

1. (A) Define an unbiased estimator of a parameter. If a statistic 'T' is an unbiased estimator for a parameter θ, then will T^2 be an UE for θ^2 ? Justify your answer.

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with finite mean μ and variance σ^2. Then :

(i) What condition must be imposed on the constants $a_1, a_2, \ldots, a_n$ so that $a_1X_1 + a_2X_2 + \ldots + a_nX_n$ is an UE for the parameter θ ?

(ii) Show that $\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ is not an UE for σ^2. Hence, obtain the UE for σ^2. 10

OR

(E) Define the following terms giving an example of each :

(i) Statistical hypothesis

(ii) Simple hypothesis

(iii) Composite hypothesis.

Also, define :

(iv) Two types of errors involved in testing of hypothesis

(v) Critical region.

What are two tailed and one tailed tests ?

Let $X \geq 1$ be the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$ on the basis of a single observation from the population, $f(x, \theta) = \theta e^{-\theta x}$, $x \geq 0$. Obtain the P[Type-I error] and P[Type-II error]. 10

2. (A) Explain the test for testing equality of variances of two normal populations on the basis of two independent small samples when :

(i) Population means are known and

(ii) Population means are unknown.

Also construct $100(1 - \alpha)\%$ confidence interval for the ratio of two population variances when population means are unknown. 10

OR

(E) Derive the test-statistic and explain the test for testing the significance of difference between two sample means based on two independent small samples drawn from normal populations. State and explain the role of F-test in carrying out the above test. 10

3. (A) Derive Brandt-Snedecor's formula for Chi-square in a $2 \times K$ contingency table.

(B) Explain the Chi-square test for testing the specified value of population variance. Also explain the construction of confidence interval for population variance. Assume that population mean is unknown. 5+5

OR

(E) Explain Chi-square test for testing the independence of attributes in $(r \times s)$ contingency table. Derive the simplified formula for chi-square statistic in (2×2) contingency table. Also, explain Yates' correction and derive simplified formula for Chi-square with Yates' correction. 10

4. (A) State Central limit theorem. Explain how central limit theorem can be used to test specified value of population mean. Also explain its use to determine confidence interval for population mean. If $X_1, X_2, \ldots, X_{64}$ are iid with mean μ and standard deviation 16, then determine the confidence interval in which μ is expected to lie almost certainly if for a particular sample $\sum_{i=1}^{64} X_i = 320$. 10

OR

(E) Explain the large sample tests for testing the equality of (i) two population means and (ii) two population proportions, by deriving the test-statistic, stating the critical region for all the three alternative hypotheses. 10

5. Solve any **ten** of the following questions :

(A) If a statistic 't' is such that $E(t) = \frac{n-1}{n-2}\theta$, where θ is a parameter, then state whether 't' will overestimate or underestimate θ. Also suggest unbiased estimator of θ in this case.

(B) State Cramer-Rao inequality.

(C) Define P-value of a test.

(D) If t follows student's t distribution and t_α is such that :

$P[|t| \geq t_\alpha] = 0.1$, state $P[t \geq t_\alpha]$ and $P[t \leq -t_\alpha]$.

(E) State the test-statistic for testing the significance of sample correlation coefficient.

(F) State $100(1 - \alpha)\%$ confidence interval for population mean based on t-statistic.

(G) Attributes 'A' and 'B' are such that, 'A' results in dichotomous classification, whereas 'B' results in manifold classification. If the degrees of freedom associated with χ^2-statistic used for testing independence of attributes is 5, how many classes does 'B' divide the population into ?

(H) State the null and alternative hypothesis of Chi-square test for homogeneity.

(I) How are degrees of freedom calculated in Chi-square test for goodness of fit ?

(J) If X_1, X_2, ..., X_n are iid with mean μ and variance σ^2, then state the condition for which $\frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ follows normal distribution :

(i) when n is small and

(ii) when X_1, X_2, ..., X_n are not normally distributed.

(K) State $100(1 - \alpha)\%$ confidence interval for population proportion.

(L) Let t denote the observed value of the test-statistic T for a test based on large sample. State the probability distribution of T and write the critical region for right tailed test. 1×10=10