Bachelor of Science (B.Sc.) Semester–IV Examination

STATISTICS

(Statistical Inference)

Optional Paper—I

Time : Three Hours] [Maximum Marks : 50

N.B. :—All questions are compulsory and carry equal marks.

1. (A) Define :

(i) An unbiased estimator

(ii) Mean squared error

(iii) Standard error

(iv) MVUE.

(B) State Cramer Rao Lower bound for variance of an estimator.

Prove or disprove :

(i) An unbiased estimator is unique.

(ii) Unbiased estimator always exists.

(iii) $2\bar{x}$ is an unbiased estimator for θ for the distribution

$$f(x) = \frac{1}{\theta}, 0 < x < \theta.$$ 10

OR

(E) Define :

(i) Simple and composite hypothesis

(ii) One tailed and two tailed hypothesis.

(iii) Power of the test.

Explain errors involved in testing of hypothesis.

Let a r.v. X have p.m.f. given below :

X :	1	2	3	4	5	6
$f_{H_0}(x)$:	1/6	1/6	1/6	1/6	1/6	1/6
$f_{H_1}(x)$:	2/15	1/6	1/5	1/5	1/6	2/15

A sample of size 1 is taken.

Let X = {2, 3} be the critical region. Find the size and power of the test. 10

2. (A) Describe the F-test for testing $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$ on the basis of two small samples from two univariate normal populations with the variances σ_1^2 and σ_2^2 respectively when means are unknown. Construct 100 (1–α)% confidence interval for the ratio of two population variances in the above case.

(B) Explain t-test for testing whether the sample mean differs significantly from the hypothetical value of the population mean, stating the assumptions clearly. Also construct 100 (1–α)% confidence interval for the population mean. 5+5

OR

(E) State the assumptions for small sample tests.

Describe the small sample test for testing the equality of population means and construct 100 (1–α)% confidence intervals for the difference of means in the following cases :

(i) Population variances are specified.

(ii) Population variances are unknown. 10

3. (A) What is meant by goodness of fit ? Explain Chisquare test for goodness of fit for a probability distribution. State the conditions for validity of Chisquare test. How are the degrees of freedom calculated ? 10

OR

(E) Derive Brandt-Snedecor formula for Chisquare in 2×k contingency table. State the formula for Chisquare in 2 ×2 contingency table. Explain Yates' correction. Why is it necessary in case of 2×2 contingency table ? Derive the modified formula for Chisquare using Yates' correction. 10

4. (A) State the assumptions for large sample tests. State central limit theorem. Explain its use in large sample tests.

Describe the large sample test for testing the significance of specified value of population mean and develop 100 (1–α)% confidence interval for the population mean, assuming that population variance is not known. 10

OR

(E) Describe the large sample test for testing the quality of two population proportions and construct 100 (1–α)% confidence interval for the difference of population proportions. 10

5. Solve any **ten** of the following :

(A) Define critical value.

(B) Define p-value and level of significance.

(C) State whether the following statement is true or false. Justify. “Type I error decreases by reducing the size of critical region.”

(D) A sample of 16 specimens taken from a normal population is expected to have a mean 50 mg/cc. The sample has a mean 64 mg/cc with a variance of 25. Write the null and alternative hypothesis to confirm the expectation. Also find value of test statistic.

(E) Write expression for pooled variance used in t-test.

(F) State a suitable test for testing the effectiveness of practising Yoga to control high blood pressure.

(G) State the test statistic used in testing the null hypothesis $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$ when population mean is specified or given.

(H) State the acceptance region in testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$.

(I) Write the null and alternative hypothesis in Chisquare test of homogeneity.

(J) State the test statistic used in testing the specified value of population proportion.

(K) State the probability distribution of the difference of sample means of two independent large samples.

(L) State 100 $(1-\alpha)$% confidence interval for the difference of population means on the basis of independent samples of large sizes if population variances are specified. 1×10=10