Bachelor of Science (B.Sc.) Semester—IV Examination

STATISTICS

(Statistical Inference)

Optional Paper—I

Time : Three Hours] [Maximum Marks : 50

N.B. :—All questions are compulsory and carry equal marks.

1. (A) Define :

(i) Unbiased estimator of parameter θ.

(i) Standard error of an estimator.

(iii) Bias in an estimator.

(iv) Root mean square error.

Prove the following :—

(i) If T is an unbiased estimator of θ, then T^2 is a biased estimator of θ^2.

(ii) If T is an unbiased estimator of θ then $aT + b$ is an unbiased estimator of $a\theta + b$ where a and b are constants.

(iii) If T_1 and T_2 are two unbiased estimators of θ and a and b are two constants then $T = aT_1 + bT_2$ is an unbiased estimator of θ if $(a + b) = 1$. 10

OR

(E) Define UMVUE. State Cramer Rao inequality. Explain its use. If a random sample is drawn from $N(\mu, 1)$ population, then using CRLB show that the sample mean is UMVUE. 10

2. (A) Describe the following tests :—

(i) paired t-test.

(ii) testing significance of sample correlation coefficient. 10

Also construct 95% confidence intervals for the difference of population means when the samples are paired.

OR

(E) Describe the tests for testing :

$$H_0 : \sigma_1^2 = \sigma_2^2 \text{ versus } H_1 : \sigma_1^2 > \sigma_2^2$$

on the basis of two small samples from two univariate normal populations with variances σ_1^2 and σ_2^2 respectively, when (i) means are known (ii) means are unknown. Construct a 100 $(1 - \alpha)$ % confidence interval for the ratio of two variances in each case. 10

3. (A) Explain the chi-square test of goodness of fit. Also explain how the degrees of freedom are determined.

(B) Describe the test for testing a specified value of population variance, when population mean is unknown. 5+5

OR

(E) State the null hypothesis and critical region of chi-square test for testing independence of two attributes. Derive Brandt—Snedecor formula of chi-square statistic in 2 × k contingency table.

(F) Explain the Chi-square test for testing homogeneity of populations. 5+5

4. (A) Explain the large sample test for testing :

(i) a specified value of a population mean.

(ii) comparison of two population means when population variances are (a) known (b) unknown. 10

OR

(E) State 'Central Limit Theorem'. Explain its use in large sample tests. Describe the large sample tests to test.

(i) Specified value of population proportion.

(ii) Significance of difference between two sample proportions. 10

5. Solve any **TEN** of the following :—

(A) Explain : The critical region at 1% level of significance is always included in the critical region at 5% level of significance.

(B) Give an example of each simple hypothesis and composite hypothesis.

(C) Define p-value of a test Statistic.

(D) The average number of articles produced by two machines per day are 200 and 250 with S.D. of 20 and 25 respectively, obtained from the record of 25 days. To test whether both the machines are equally efficient, state the null and alternative hypothesis that should be tested.

(E) Let $F_{(m,n)}$, α denote the value of $F_{(m,n)}$ such that $P[F \geq F_{(mn),\alpha}] = \alpha$. Then show that :

$$F_{(n, m),(1-\alpha)} = \frac{1}{F_{(m, n), \alpha}}.$$

(F) Explain the need of Yates' correction in 2 × 2 contingency table.

(G) Obtain 95% confidence interval for the population mean μ, given that a random sample of size 100 drawn from a normal population $(\mu, 1)$ gave $\bar{x} = 12{\cdot}60$. ($Z_{0.025} = 1.96$)

(H) In tossing a coin 100 times, a boy gets head 66 times. To test whether the coin is unbiased, which test should be used ? Write its test–statistic.

(I) Which test is always a right tailed test ?

(J) State 100 (1 – α)% confidence interval for population mean based on small sample, when population variance is unknown.

(K) State the test statistic used for testing the specified value of variance of normal population, on the basis of small sample, when population mean is known.

(L) State 100 (1 – α)% confidence interval for a specified value of a population mean, when the sample is large. 1×10=10